The Supply of Money under Asymmetric Information: Monopoly versus Competition*  

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Abstract  

I study the efficiency of issuing asset-backed money under asymmetric information by comparing a competitive setting with a monopoly. In the model bankers can issue money by holding a real asset, but they can also fake the quality of the asset at a proportional cost. Therefore, when the counterfeiting cost is low, a haircut arises in the money transactions and the aggregate liquidity can be dried up in equilibrium. This competitive equilibrium is sub-optimal, because the individual issuers do not internalize the effect of issuing money on the prices and the haircut in general equilibrium. A concentrated banking system such as a monopoly can be an alternative way to improve welfare. Despite the inefficiency from the monopoly rent, the monopoly issuer can adjust the money supply by considering the effects on prices and its own counterfeit incentive. Imposing reserve requirements is also effective to raise the transparency of the banker’s asset holdings and restore the constrained efficiency in the competitive equilibrium.  

JEL Codes: E42, E58, G21, G38  

Key words: Limited commitment, Moral hazard, Transparency, Market Structure, Reserve requirements.  

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1 Introduction

In most modern economies, money is supplied by central banks instead of markets. However, before the U.S. Civil War the dominant medium of exchange in the U.S. was private bank notes. From 1837 to 1862 there was no formal central bank and a number of small banks issued their own bank notes competitively, which circulated as money [Schuler (2001)]. These bank notes traded at a discounted value by reflecting the issuing banks’ assets [Rockoff (1974), Rolnick and Weber (1983)], and newly launched banks’ liabilities were discounted more heavily [Gorton (1996)]. When there is no doubt on its value, money is accepted at par, so banks tend to keep the information on their assets secret to improve liquidity creation. However, there were a series of financial panics until early 1900s because the banks are subject to bank runs at a cost for this opacity [Gorton (2013)].

In the recent Great Recession of 2007-2009 a similar type of failure has been repeated in the financial market, where asset-backed securities issued by banks were used to facilitate transactions among the financial intermediaries. Since it is hard to verify the quality of these banks’ asset-holdings, the haircut of the asset-backed securities for the collateral transactions rose sharply, so the liquidity in the financial market was rapidly dried up during this period.¹ These two historical facts raise some questions that I wish to address in this paper. Is the market-based liquidity provision efficient under asymmetric information? If it is not, what types of regulations are required? Can we consider a concentrated banking system such as a monopoly or an oligopoly in money supply as an alternative?

For the optimal market structure for money supply, many economists argued that private monetary systems are feasible and can be self-regulated, based on some successful cases of the laissez-faire banking system in history.² On the other hand, Friedman and Schwartz (1986) suggest several reasons why monetary system cannot be left to the market.³ Nevertheless, there are not many theoretical models developed to study this issue in a respect of opacity. In this paper I focus on collateral misrepresentation incentives of the liquidity providers and study how this incentive problem can create an inefficiency in the private monetary system.

In order to explore this issue I modify a Lagos and Wright (2005) type monetary model by introducing a real asset and durable good consumption from Velde and Weber (2000) and counterfeit incentives from Li et al. (2012). This micro-founded model has an advantage of incorporating informational frictions such as lack of memory, moral hazard and limited commitment in

¹See Gorton and Metrick (2012), Guerrieri and Shimer (2014) for more details.
²See Klein (1974), Hayek (1974), King (1983), and Calomiris and Kahn (1996) for the arguments. Rolnick and Weber (1983) discuss that unfettered banking systems in some states such as New York in Free Banking Era are well-managed, while Smith and Weber (1999) and Cowen and Kroszner (1989) evaluate the Suffolk Banking system in U.S. in 1824-1858 and the Scottish free-banking system in 19th century as well-functioned private monetary systems, respectively.
³They summarize the reasons as the resource cost, the risk of fraud, the capability of controlling quantity, and the presence of externality. See Selgin (2008) for more details.
a simple way. The main features of the model are as follows. Agents can produce consumption goods with an elastic labor supply, but cannot consume their own output. To trade with each other, agents need to use a medium of exchange, money, but under limited commitment money must be backed by the asset holdings of the third party, referred as bankers. Since money is just a paper claim, there could be moral hazard incentives of the bankers to hold less amount of the asset. In the model, the individual bankers can create a fake asset at a proportion cost to issue money. Therefore, agents can recognize what proportion of money transactions is secured by the genuine asset holdings given the prices of money and the asset. The asset can be used not only for supporting money issuance, but also for durable goods consumption of the agents. Thus, given a fixed supply of the asset, the price of the asset is determined by the demands for issuing money and consuming durable goods in the competitive market.

In the model a non-monetary equilibrium can arise when the cost for faking collateral is sufficiently low. The asset cannot be used for durable good consumption when it is held for supporting money transactions. Thus, when the counterfeiting cost is less than the marginal utility of the asset for durable good consumption, the sellers recognize that money could be backed by a fake asset, so they will not accept money. This implies that if an intrinsic value of an asset is high, then it might not be suitable to use the asset as collateral under the opacity. This finding could provide one way to explain the reason why we had moved from asset-backed money to fiat money in history.

In competitive equilibrium, the bankers provide money until the marginal benefit of issuing money is equal to the opportunity cost of holding the asset. If the cost of faking the asset is sufficiently high, the equilibrium allocation is efficient in a respect of liquidity provision. However, a liquidity dry-up can happen when the cost of faking the asset is low enough. As the incentive for counterfeiting becomes severe, the more quantity of the asset is required to support the same unit of real money transactions. Therefore, both prices of the asset and money rise because the bankers want to purchase more of the asset for earning profits, and the supply of money becomes more scarce. Then, the required quantity of asset for issuing money will be much greater because everyone knows that the higher prices of the asset and money would encourage the bankers to fake the asset further. Hence, the supplies of money and the asset in the market would be even more scarce by this amplification mechanism. This competitive equilibrium with a liquidity dry-up is sub-optimal because of the pecuniary externality. The individual bankers are price-takers.

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4In the previous literature such as Nosal and Wallace (2007), there also exists a non-monetary equilibrium when money can be counterfeited. However, this non-monetary equilibrium result in my model depends on the asset price. Moreover, each non-monetary and monetary equilibrium is unique in my model since the asset has a strictly positive intrinsic value for durable good consumption. Therefore, this non-monetary equilibrium is different from the multiple equilibria in the models with fiat money such as Wallace (1978) and Lagos and Wright (2003), where money is not accepted in a self-fulfilling inflationary equilibrium.

5Unlike Lagos and Rocheteau (2008), the fixed supply of real asset is assumed to be scarce for monetary transactions in the model. Thus, as much as the asset is used for liquidity, the asset price rises and welfare improves.
so given prices, they keep providing money until there is no profit. Since the prices of money and
the asset are determined in the competitive market, the individual bankers cannot internalize the
impact of their money issuance on the prices and their counterfeit incentives in general equilib-
rium. Therefore, when the incentive constraint binds, the liquidity dry-up is amplified, and thus
the social loss from the pecuniary externality becomes greater.

A concentrated banking system can be an alternative mechanism in this case. If only one is-
suer is authorized by the government to issue money, this monopoly issuer will adjust the money
supply to maximize his/her profit. Therefore, a social loss associated with the monopoly rent is in-
etable. However, the monopoly issuer is a price-maker, so he/she can internalize the effect of the
aggregate money supply on prices. This single issuer would choose the money supply and prices
to discourage his/her faking incentive, because it allows him/her to reduce the cost of providing
money and increase his/her profit. Therefore, when the cost of faking the asset is sufficiently low,
welfare of monopoly equilibrium can be greater than that of competitive equilibrium.6

Additionally, I consider two types of regulations such as reserve requirements and an entry
barrier to understand how these regulations can correct the inefficiency in the competitive equi-
librium. Imposing a reserve requirement is effective to restore the constrained efficiency when the
liquidity is dried up by the amplification mechanism. Although the reserves are not actually used
as a buffer in a liquidity crisis, this requirement can increase the proportion of the asset which
cannot be faked. Thus, once the price of the asset is lowered with a small amount of the genuine
asset, the inefficiency can be also eliminated by the amplification. An entry cost is also effective to
raise the transparency of the bankers since it requires the bankers to earn a strictly positive profit.
However, it is relatively limited since it cannot unwind the liquidity dry-up.

These findings can provide a new insight for the transparency of the financial intermediaries.
One of their primary functions is to create and provide the liquidity by issuing debts, since their
liabilities such as deposit claims and asset-backed securities are used as means of payment in the
financial market. If it is costly to verify the quality of their asset portfolio, then the less desirable
outcome can be addressed by competition. In this respect, the transparency of the banking sec-
tor is important not only for the financial stability, but also for the efficient liquidity provision.
Moreover, this paper provides a rationale for reserve requirements on the financial intermedi-
aries. However, the main reason is different from the literature which studies reserve requirements
in a respect of the liquidity insurance. In my paper reserve requirements play a role for raising the
transparency of the banks rather than responding to the liquidity demand.

6This point is different from some papers such as Hellmann et al. (2000) and Christiano and Ikeda (2016) which
emphasize the franchise value of the monopoly supplier to deter the moral hazard incentives, because in my model
the monopolistic equilibrium could be better off even though the equilibrium profit is sufficiently small in my model.
Moreover, this idea is close to a general literature including Spence (1975), Musa and Rosen (1978), and Judd and
Riordan (1994) which studies extensively how a monopoly can be better when the firm can choose both quantity and
quality of the good.
Finally, I extend the model by introducing unsecured debt and confirm that there still exists a set of parameters in which monopolistic equilibrium is better than competitive one. As long as the bankers can issue asset-backed money, they purchase the asset and issue money until their profits approach to zero in equilibrium. Therefore, the inefficient liquidity dry-up allocation remains in competition. On the other hand, in case of monopoly, the single supplier will not buy the asset, since it is costly. He/she will issue money supported only by unsecured debt as shown in Sanches (2016). Therefore, the monopolistic allocation can be even better than the allocation which relies only on secured debt.

1.1 Related Literature

After the seminal contribution of Gorton and Pennacchi (1990), there is a vast literature which studies the liquidity creation function of the banks in a theoretical approach.\(^7\) Among them, this paper is clearly related to the papers that show an inefficiency of liquidity provision in a competitive banking system and suggest a concentrated one as an alternative. Gersbach (1998) points out that there exists a negative externality of issuing banknotes because other banks require to invest more in short-term assets in order to satisfy liquidity needs. Boyd et al. (2004) focus on the liquidity insurance of banks and show that the monopoly bank can be less fragile. Although the monopoly bank hold less reserves, the probability of crisis can be reduced because the rate of return on depositors becomes lower in monopoly. In my paper I also compare the competitive setting with a monopoly, but I concentrate on the aggregate supply of liquidity rather than the fragility of the banks.

In this respect my paper is closely related to the literature on the aggregate supply externality in the private liquidity creation. Hart and Zingales (2011) find out an interesting pecuniary externality in money supply. In their model the price of money falls as more money is supplied. Therefore, money can be over-produced in general equilibrium.\(^8\) In this paper I find out that there can be a pecuniary externality in money supply associated with asymmetric information on the liquidity provider. In the model when the pledgeability is less than one, money requires more assets to be backed, so the total supply of liquidity can be reduced. Since this pledgeability is endogenously determined by the price of the asset in a way of reflecting the moral hazard incentive, there arises a pecuniary externality in money supply.

Another important papers that study the inefficiency of competitive money issuance is Monnet and Sanches (2015) and Sanches (2016), which emphasize the franchise value of the banks. Monnet and Sanches (2015) show that when the asset portfolio is unobservable, a positive franchise value

\(^7\)See Bouwman (2018) for a literature review on liquidity creation of banks.

\(^8\)Recently, Benigno and Robatto (2019) study the effect of the risk-debt issuance and show that the aggregate liquidity can be reduced by a default. Luck and Schempp (2019) show that an endogenous fire-sale risk can reduce the aggregate liquidity creation.
of banks is required to expand the limit of their money issuance. Sanches (2016) finds out that the self-fulfilling collapse is feasible in a private monetary system when money is backed by the future seignoirage, and suggest a monopolistic supplier as an alternative. In my paper I also compare the competitive and monopoly regimes, but I focus more on asset-backed money and how the bankers can use asset prices to reveal their transparency under asymmetric information.

The search-based approach on the recognizability of money is pioneered by Williamson and Wright (1994). They show that when the commodity are not recognizable, bad commodity can be produced, and accepted with a positive probability, which is sub-optimal. Therefore, in their model introducing fiat money, which cannot be produced privately, can encourage people to produce and trade good commodities. In my paper the bankers can produce fake collateral, but it is a threat and will not happen in equilibrium. Moreover, unlike their papers, if agents recognize that money can be backed by fake commodity, they will not accept money in my model. My paper is also related to Lester et al. (2012), in which the recognizability of one asset is endogenously determined by investing the cost-veriﬁcation technology. In their paper when the cost of faking decreases into zero, the probability of acceptance can go to zero. Unlike their paper, in my paper although the proportional cost for faking is positive, a non-monetary equilibrium can happen because the pledgeability could be zero when the faking cost is lower than the asset’s intrinsic value. Therefore, while those papers are silent in the impact of the prices on the information friction, I focus on how the pledgeability of assets is determined endogenously along with the asset prices by reflecting the fake incentives.

This paper is also related to the literature on the opacity of the liquidity provider’s assets. Kaplan (2006), Andolfatto et al. (2014), and Dang et al. (2017) show that limited disclosure is beneficial because of pooling investment risks, whereas Alvarez and Barlevy (2015) find out that disclosure of bank loss can reduce the social cost associated with contagion. While these papers focus on revealing the value of risky assets, I study how to show transparency by adjusting the price of assets given the moral hazard incentive. In a perspective of modelling this incentive to fake collateral is based on the framework of Li et al. (2012) and Williamson (2018), who study the transactions in over-the-counter market and the effect of private asset purchase under the threat of faking assets, respectively. Holmstrom and Tirole (1998) and Hellmann et al. (2000) also have the moral hazard incentive of the banks in their models, but they focus more on the risk-taking behavior rather than the collateral misrepresentation.

Finally, empiric work on the relationship between liquidity creation and market structure is scant. Berger and Bouwman (2009) construct a measure of liquidity creation and evaluate the effects of bank size, bank capital on the liquidity creation. Recently, Jiang et al. (2016) find out that banks become more transparent under competition, because banks make more effort on monitoring and screening ﬁrms. Their result encourages the information disclosure of the banks, but my paper studies how to control the moral hazard incentive without the information disclosure.
2 The environment

The basic structure of the model is related to Rocheteau and Wright (2005) in which ex ante heterogeneous agents trade goods with money in decentralized meetings and adjust their asset portfolio in the competitive markets. Time \( t = 0, 1, 2, \ldots \) is discrete and the horizon is infinite. Each period is divided into two sub-periods - centralized meetings (CM) followed by decentralized meetings (DM).

There is a continuum of buyers, sellers, and bankers, each with a unit mass. An individual buyer has preferences

\[ E_0 \sum_{t=0}^{\infty} \beta^t [-H_t + u(x_t) + v(a_t)], \]

where \( H_t \in \mathbb{R} \) is the labor supply of the buyer in the CM, \( x_t \in \mathbb{R}_+ \) is the DM good consumption of the buyer in the DM, and \( 0 < \beta < 1 \). Assume that \( u(\cdot) \) and \( v(\cdot) \) are strictly increasing, strictly concave, and twice continuously differentiable with \( u'(0) = v'(0) = \infty, u'(\infty) = v'(\infty) = 0, -xu''(x) u'(x) = \sigma_u < 1, \) and \(-av'(a) v'(a) = \sigma_v < 1\). Each seller has preferences

\[ E_0 \sum_{t=0}^{\infty} \beta^t [X_t - h_t], \]

where \( X_t \in \mathbb{R} \) is the CM good consumption of the seller in the CM, and \( h_t \in \mathbb{R}_+ \) is the labor supply (or the DM good production) of the seller in the DM. An individual banker has preferences

\[ E_0 \sum_{t=0}^{\infty} \beta^t [X^b_t - H^b_t], \]

where \( H^b_t \in \mathbb{R}_+ \) is the labor supply of the banker in the CM, and \( X^b_t \in \mathbb{R}_+ \) is the CM good consumption of the banker in the CM. All the agents can consume and produce in the CM. But in the DM only buyers can consume and only sellers can produce. One unit of labor input produces one unit of perishable consumption good either in the CM or in the DM.

In the DM each buyer meets one seller, and vice-versa, bilaterally. The terms of trade are determined by bargaining between the buyer and the seller. For simplicity, it is assumed that the buyer has all bargaining power. There is no record-keeping technology for buyers and sellers. Also, under limited commitment no one can be forced to work. Thus, recognizable assets are essential for trade in the DM, and the trade must be quid pro quo.

There is a real asset in this economy with one unit of fixed supply. This real asset can be used for durable good consumption of the buyer or held by the banker to support money, but cannot be used for transactions directly.\(^9\) The bankers cannot participate in the DM trade, but they can

\(^9\)At least \( xu'(x) \) must increase in \( x \) to obtain a steady state equilibrium in the model.

\(^10\)In my paper the real asset represents commodities, real estate, and equipment in reality, which are less preferred in transactions. Since the values of these assets are verified only by the experts such as banks, I assume that buyers and
provide information such as their names, addresses and asset holdings to the buyers and sellers. The bankers can issue their monies by purchasing the asset in the CM, and the issued monies can be used as a medium of exchange in the DM meetings.\textsuperscript{11} The money can circulate among buyers and sellers over time, but if requested the bankers need to pay a promised amount of the asset to the holder of money in the CM. Under limited commitment, if the bankers refuse to redeem the money in the next CM, the underlying asset would be seized and transferred to the holder of the money, but the banker can keep issuing money by purchasing the asset in the next period.

In order to incorporate asymmetric information, I assume that the bankers can also provide money by producing fake assets at a cost of $\gamma > 0$ per unit of assets in terms of the CM good.\textsuperscript{12} This collateral misrepresentation assumption captures the opacity of the banks' balance sheets. In equilibrium this counterfeiting behavior will not occur, but the bankers cannot support the money transactions up to the full value of their asset holdings since they cannot show their honesty. Thus, only a pledgeable proportion, $\theta_t \in (0, 1]$, of the underlying asset is used as collateral to support the issued money. One unit of the asset is traded at price $\psi_t \in \mathbb{R}_+$ and money sells at price $q_t \in \mathbb{R}_+$ in units of the CM good in the period $t$ CM.

![Figure 1: Transaction Process](image)

Timing is as follows. In the beginning of period $t$ CM, all agents meet together. The previous obligations such as money are paid off with the labor supply or promised asset. After a competitive asset market opens, CM goods are produced, the asset is traded, the bankers issue their own monies, and the buyers purchase money. In the DM, buyers and sellers meet bilaterally and trade with money. In the period $t + 1$ CM, the sellers redeem money to the banker and/or sell the received money holdings in the competitive asset market as described in Figure 1.

\textsuperscript{11}These monies are perfect substitutes as long as they are well supported by the asset.

\textsuperscript{12}It is assumed that money itself cannot be counterfeited.
3 Competitive Banking System

3.1 Maximization Problems

In the model we have two maximization problems, one for the bankers and the other for the buyers, because the seller’s decision is made passively with no bargaining power.\textsuperscript{13} The bankers can provide a payment instrument, money, for the DM trade, but they are required to hold the genuine asset as collateral and/or produce a fake asset to support the money transactions under limited commitment.

In the model an individual banker solves the following problem in the CM of period $t$:

$$\max_{a_t, m_t \geq 0} \Pi_t := -\psi_t a_t^i + \beta \psi_{t+1} a_t^i + q_t m_t - \beta m_t$$

subject to a collateral constraint,

$$\beta \psi_{t+1} a_t^i \theta_t - \beta m_t \geq 0$$

and an incentive constraint (IC),

$$-\gamma a_t^i + q_t m_t \leq -\psi_t a_t^i + \beta \psi_{t+1} a_t^i + q_t m_t - \beta m_t.$$  \hspace{1cm} (3)

In (1)-(3), $a_t^i \in \mathbb{R}_+$ denotes the quantity of the asset purchased by the banker and $m_t \in \mathbb{R}_+$ denotes the supply of money in units of the CM good. This money is a promise to pay $m_t$ units of the CM good in the period $t + 1$ CM, and sells at the price $q_t$, in units of the CM good in period $t$. The representative banker maximizes its profit, $\Pi_t$, in the objective function (1), which includes the net payoff on the purchase of the asset and the revenue from the issuance of money. Eq. (2) is a collateral constraint for the banker, which states that the payoff on the money in the period $t + 1$ CM cannot exceed the pledgeable proportion, $\theta_t$, of the total value of the asset held by the banker. Eq. (3) is a counterfeit incentive constraint (IC) for the banker, in which the banker can create fake assets as collateral at a proportional cost, $\gamma$, to issue money.

In the model we have another maximization problem for the buyers. The buyers want to hold money for the DM transactions and purchase the asset for durable good consumption. A representative buyer solves the following problem in the CM of period $t$:

$$\max_{m_t, a_t, x_t \geq 0} - q_t m_t + u(x_t) - \psi_t a_t + \beta \psi_{t+1} a_t + v(a_t)$$

subject to the seller’s participation constraint,

$$\beta m_t - x_t \geq 0.$$  \hspace{1cm} (5)

\textsuperscript{13}Since a seller can only choose whether to accept or reject, a buyer always suggests an offer which is accepted.
In (4)-(5), \( x_t \in \mathbb{R}_+ \) denotes the quantity of the DM good that the buyer consumes in the DM meeting. The variables \( m_t \in \mathbb{R}_+ \) and \( a_t \in \mathbb{R}_+ \) are the quantities of money and the asset in terms of CM good purchased by the representative buyer. The maximization problem (4) subject to (5) states that the buyer maximizes the expected value subject to the seller’s non-negative net payoff constraint (5).

Given the unit supply of the asset, the first-best allocation includes the maximum asset consumption, \( a_t = 1 \), and the efficient amount of transactions in the DM, \( x_t = x^* \) where \( u'(x^*) = 1 \). Note that this first-best allocation is infeasible in the model because the amount of transactions, \( x_t \), must be supported by the bankers’ asset holdings, \( a'_t \), and the total supply of the asset is limited as one.

### 3.2 Competitive Equilibrium

Under perfect competition a continuum of bankers are competitive to issue money by purchasing the asset. Given the scarcity of the total asset, the collateral constraint (2) always binds in equilibrium. Therefore, by plugging (2) into the IC (3), the pledgeable proportion, \( \theta_t \), can be written as

\[
\theta_t = \min \left[ 1, \max \left\{ \frac{\gamma - \psi_t + \beta \psi_{t+1}}{\beta \psi_{t+1}}, 0 \right\} \right],
\]

which cannot exceed one and must be non-negative. Notice that \( \theta_t = 1 \) when \( \gamma \geq \psi_t \), and \( \theta_t = 0 \) when \( \gamma \leq \psi_t - \beta \psi_{t+1} \), and \( \theta_t \in (0,1) \) when \( \psi_t - \beta \psi_{t+1} < \gamma < \psi_t \) in (6). After plugging (3) and (6) into (1), the first-order condition of the bankers for the asset, \( a'_t \), can be derived as

\[
(a'_t) \quad \psi_t = \beta \psi_{t+1} + (q_t - \beta) \psi_{t+1} \theta_t = \beta \psi_{t+1} + \min \left[ (q_t - \beta) \psi_{t+1}, (1 - \frac{\beta}{q_t}) \gamma \right].
\]

Note that if \( \theta_t = 1 \) then the rates of return on money and the asset are equal, \( \frac{1}{q_t} = \frac{\psi_{t+1}}{\psi_t} \) in (7). On the other hand, if \( \theta_t < 1 \) then the rate of return on money, \( \frac{1}{q_t} \), must be smaller than the rate of return on the asset, \( \frac{\psi_{t+1}}{\psi_t} \): since it is required to hold more asset for issuing the same unit of money, the bankers reduce the rate of return on money to earn more seigniorage per unit of money.

From the buyer’s maximization problem we can have the first-order conditions for \( m_t, a_t \), respectively, as

\[
(m_t) \quad q_t = \beta u'(x_t),
\]

\[
(a_t) \quad \psi_t = \beta \psi_{t+1} + v'(a_t).
\]

In (8)-(9) the current prices of money and the asset are equal to the present value of the marginal utility of money trade, and the resale price in the next period plus the marginal utility of the durable good consumption, respectively.
In equilibrium asset markets clear in the CM for all time \( t \), so the demands of the issuers and buyers for money and the asset are equal to the supplies of money and the asset, respectively, as
\[
a_t + a'_i = 1, \quad m_t = m_t. \tag{10}
\]
\[
\text{Definition 1.} \text{ Given } \gamma, \text{ a competitive equilibrium is a sequence of the quantities } (a_t, x_t, m_t), \text{ the prices } (\psi_t, q_t), \text{ and the pledgeability } \theta_t, \text{ which satisfy equations (2),(5)-(11) at each period } t.
\]

### 3.3 Steady States

In this subsection we characterize the steady states by whether the IC (3) binds or not. Since we focus on the stationary equilibrium allocation, the time scripts \( t \) in the variables are removed in this subsection. By plugging the buyer’s first-order conditions (8)-(9) into the issuer’s first-order condition (7), we can have a zero-profit condition,
\[
\theta \{ u'(x) - 1 \} = \frac{1}{\beta} - 1, \tag{12}
\]
which represents that the marginal benefit of issuing money by purchasing one unit of asset, \( \theta \{ u'(x) - 1 \} \), is equal to the marginal cost of holding one unit of asset without using it for durable good consumption, \( \frac{1}{\beta} - 1 \). Moreover, given the scarcity of the asset supply, the constraints (2) and (5) always bind in equilibrium, thus we can obtain,
\[
x = \beta \bar{m} = \beta \psi a^i \theta. \tag{13}
\]

By plugging (13) and the buyer’s first-order conditions (9) into (10), we can have an equilibrium condition for feasibility,
\[
\frac{x}{\beta \psi \theta} + a = \frac{(1 - \beta)x}{\beta \psi'(a) \theta} + a = 1, \tag{14}
\]
where \( \psi = \frac{v'(a)}{1 - \beta} \) holds from (9).

(i) **Case with** \( \theta = 0 \): If \( \gamma \) is sufficiently small as \( \gamma \leq (1 - \beta) \psi \), then \( \theta = 0 \) holds in (6). In this case, for any \( a', x = 0 \) holds inevitably in (13), so the banker will not hold the asset to issue money. Thus, there exists only a non-monetary equilibrium with \( x = 0 \) and \( a = 1 \), and the price of the asset will be \( \psi = \frac{v'(1)}{1 - \beta} \) in equilibrium. Hence, this equilibrium allocation can be supported when \( \gamma \leq (1 - \beta) \psi = v'(1) \).

(ii) **Case with** \( \theta = 1 \): If the IC (3) does not bind, then \( \theta = 1 \). Thus, \( x = \bar{x} \) where \( u'(\bar{x}) = \frac{1}{\beta} \) in (12), and \( a = \bar{a} \) is uniquely determined from (14) with \( \theta = 1 \). There is no discount in the price
of money, \( q_c = 1 \) in (8) with \( u'(\bar{x}_c) = \frac{1}{\beta} \), because money is equally valuable as the asset. In order to support this equilibrium allocation \((\bar{a}_c, \bar{x}_c)\) with \( \theta = 1 \), \( \gamma \geq \bar{\psi}_c \frac{v'(a_c)}{1-\beta} \) is required from (6). Thus, when \( \gamma \) is sufficiently large, i.e. \( \gamma \geq \bar{\psi}_c \), the incentive constraint (3) does not bind and the equilibrium allocation is determined at \((\bar{a}_c, \bar{x}_c)\).

(iii) Case with \( \theta \in (0, 1) \): If the IC (3) binds with \( \theta \in (0, 1) \), then by plugging (6) into (12) and (14), the equilibrium conditions (12) and (14) can be rewritten as

\[
\gamma \frac{u'(x)}{u(x)} = \gamma - v'(a),
\]

(15)

and

\[
x = \{\gamma - v'(a)\}(1 - a),
\]

(16)

respectively. The zero-profit condition (ZC) with \( \theta \in (0, 1) \) (15) shows a strictly positive relationship between \( a \) and \( x \), while the feasibility condition (FC) with \( \theta \in (0, 1) \) (16) is hump-shaped as described in Figure 2 unlike the FC with \( \theta = 1 \) (14). If the IC (3) does not bind with \( \theta = 1 \), when the bankers purchase more asset, \( a' \), the money transaction, \( x \), increases in (13). However, if the IC (3) binds, then the pledgeability, \( \theta \), is endogenously determined along with the price of the asset, \( \psi \). Specifically, the pledgeability, \( \theta \), decreases in the price of the asset, \( \psi \), in (6): when the price of the asset, \( \psi \), rises, the bankers prefer to produce the fake asset because it is more costly to purchase the genuine asset. Therefore, if the pledgeability \( \theta \) decreases much more than the rise in the asset price, \( \psi \), then the money transactions, \( x \) can decrease in (13).

![Figure 2: Competitive Equilibrium](image)
Lemma 1. When $\theta \in (0, 1)$, the stationary monetary equilibrium allocation $(\hat{a}_c, \hat{x}_c)$ is uniquely determined by (15)-(16).

Proof. See the appendix.

In Figure 2, points A, B and C represent the allocations at $\theta = 0$, $\theta = 1$, and $\theta \in (0, 1)$, respectively.\footnote{Note that (15)-(16) have another intersection, $x = 0$ and $a = \hat{a}$, but it is not an equilibrium allocation since the profit is strictly negative.} Notice that the FC with $\theta \in (0, 1)$ (16) is dominated by the FC with $\theta = 1$ (14) because for the same asset-holdings, $a^i$, the money trade, $x$, is always smaller due to $\theta < 1$.\footnote{Since the feasibility condition (16) is hump-shaped, the equilibrium allocation is feasible only at $a \in (\hat{a}, 1)$ where $\hat{a}$ is defined by $\gamma = \hat{v}'(\hat{a})$ in Figure 2.}

Moreover, $\hat{x}_c < \bar{x}_c$ since $\theta < 1$ in (12), and so $\hat{q}_c = \beta \hat{u}'(\hat{x}_c) > 1 = \bar{q}_c$ in (8). It implies that when the IC (3) binds with $\theta < 1$, the more asset is required to issue one unit of money, so the money supply is reduced and the price of money rises. Actually, the decline in the money supply is amplified as the price of the asset rises further: once the IC binds, more asset is required to support the same amount of money. Then, the price of the asset rises, so the pledgeability decreases further. Hence, the money supply could be restricted furthermore. In order to support this equilibrium allocation $(\hat{a}_c, \hat{x}_c)$ with $\theta \in (0, 1)$, $\hat{v}'(1) < \gamma < \bar{\psi}_c$ is necessary to avoid either $\theta = 0$ or $\theta = 1$. Finally, we can summarize the equilibrium allocations by $\gamma$ as follows.

Lemma 2. Given $\gamma$, the following hold:

(a) If $\gamma \leq \hat{v}'(1)$, there exists only a non-monetary stationary equilibrium with $x = 0$ and $a = 1$.

(b) If $\hat{v}'(1) < \gamma < \bar{\psi}_c$, there is a unique steady state with $(\hat{a}_c, \hat{x}_c)$.

(c) If $\gamma \geq \bar{\psi}_c$, there is a unique steady state with $(\bar{a}_c, \bar{x}_c)$.

Proof. See the appendix.

3.4 Dynamics

In this subsection we analyze the dynamics of the asset price to check the stability of the steady states and to understand the equilibrium characteristics in depth. I assume the following utility functions to address it formally.

Assumption 1. Assume $u(x) = \frac{x^{1-\sigma_u}}{1-\sigma_u}$ and $v(a) = \frac{a^{1-\sigma_v}}{1-\sigma_v}$ with $(\sigma_u, \sigma_v) \in (0, 1) \times (0, 1)$.

(i) Case with $\theta_t = 0$: If $\theta_t = 0$ at time $t$, then $a_t = 1$. Therefore, the asset pricing equation (9) can be rewritten as

$$\bar{\psi}_t = \beta \bar{\psi}_{t+1} + \hat{v}'(1).$$
Note that this dynamic equation (DE) (17) can be applied only when \( \gamma \leq v'(1) \), since \( \theta = 0 \) holds when \( \gamma \leq \psi_t - \beta\psi_{t+1} \) from (6).

(ii) Case with \( \theta_t = 1 \): If the IC (3) does not bind, by plugging all the equilibrium conditions (2),(5),(8)-(11) with \( \theta = 1 \) into (7), we can obtain a dynamic equation for the price of the asset as

\[
\psi_t = \beta\psi_{t+1} + \left[ 1 - (\beta\psi_{t+1})^{\frac{1}{\sigma_u}} - \frac{1}{\sigma_u} \right]^{1 - \sigma_v}.
\] (18)

Note that when \( \psi_{t+1} \) approaches to zero, \( \psi_t \) is close to \( v'(1) \). Moreover, the slope of this dynamic equation is

\[
\frac{\partial \psi_{t+1}}{\partial \psi_t} \bigg|_{DE \theta = 1} = \frac{1 + G(\psi_t, \psi_{t+1})}{\beta \{ 1 + G(\psi_t, \psi_{t+1}) \}}
\] (19)

where \( G(\psi_t, \psi_{t+1}) := \frac{\sigma_v}{\sigma_u} \left\{ 1 - (\beta\psi_{t+1})^{\frac{1}{\sigma_u}} - \frac{1}{\sigma_u} \right\}^{1 - \sigma_v} - (\beta\psi_{t+1})^{\frac{1}{\sigma_u}} - \frac{1}{\sigma_u} \). This DE (18) is strictly convex, because the slope increases in (19) as \( \frac{\psi_{t+1}}{\psi_t} \) increases. Additionally, the slope is strictly positive, \( \frac{\partial \psi_{t+1}}{\partial \psi_t} > 0 \), for all \( \psi_t \) and if \( \frac{\psi_{t+1}}{\psi_t} = \frac{1 - \sigma_v}{\beta} \), then the slope is equal to the inverse of time preference, \( \frac{\partial \psi_{t+1}}{\partial \psi_t} = \frac{1}{\beta} \). Finally, this equation can be applied only when \( \gamma \geq \psi_t \) from (6).

(iii) Case with \( \theta_t \in (0, 1) \): If the IC (3) binds with \( \theta_t \in (0, 1) \), then by using (6) we can obtain a dynamic equation,

\[
\psi_t = \beta\psi_{t+1} + \left[ 1 - (\gamma - \psi_t + \beta\psi_{t+1})^{\frac{1}{\sigma_u}} - \frac{1}{\sigma_u} \right]^{1 - \sigma_v}.
\] (20)

In this case the slope in (20) is constant as

\[
\frac{\partial \psi_{t+1}}{\partial \psi_t} \bigg|_{DE \theta \in (0,1)} = \frac{1}{\beta'}
\] (21)

and this dynamic equation (20) can be applied only when \( \psi_t - \beta\psi_{t+1} < \gamma < \psi_t \) from (6).

Figure 3: Dynamic Equations in Competitive Equilibrium
We can describe the dynamic equations (17)-(18) and (20) by the level of $\gamma$ in Figure 3. In panel (a) if $\gamma \leq v'(1)$, the DE (17) is applied for all $\psi_t$, since $\gamma \leq \psi_t - \beta \psi_{t+1}$ always holds from (17). On the other hand, if $\gamma > v'(1)$, then the DE (18) is applied for $\psi_t \leq \gamma$, while the DE (20) is applied for $\psi_t > \gamma$ as shown in panels (b) and (c) in Figure 3. Notice that each point A, B and C in Figure 3 corresponds to each steady state point A, B and C in Figure 2. Since the dynamic equations (17)-(18) and (20) are all strictly increasing, the steady state equilibrium is unique for each case. Moreover, the equilibrium allocations at the points A, B and C are all stable because there is no equilibrium with a self-fulfilling collapse.\footnote{There is no self-fulfilling collapse as shown in Sanches (2016), because the asset has always an intrinsic value as long as it can be used for durable good consumption.}

Note that this non-monetary equilibrium result with $\gamma \leq v'(1)$ is different from the previous literature on counterfeiting fiat money: the money trade can be zero although the proportional cost is still strictly positive. Since the asset can be also used for durable good consumption instead of backing money, the marginal cost of using the genuine asset will be equal to this intrinsic value. Therefore, whenever the counterfeiting cost is lower than this intrinsic value, i.e. $\gamma \leq v'(1)$, then it is believed that the money is backed by nothing. More importantly, as this intrinsic value of the asset, $v'(1)$, increases, it is more likely to shut down the money transactions. This result is somewhat ironic since as more as the asset is more valuable, it should not be used for supporting the money trade. However, it is reasonable under the counterfeiting incentives, because it is more profitable to fake collateral as much as the backed asset are demanded for other usages.
\( v'(1) < \gamma < \bar{\psi}_c \), by using the DEs (18) and (20) we can describe the steady state asset price \( \psi \) according to \( \gamma \) as shown in Figure 4. Since the slopes of DEs (18) and (20) are equal when \( \frac{\psi_{t+1}}{\psi_t} = \frac{1-c_u}{p} \), if \( \frac{1-c_u}{p} < 1 \), then there exists a maximum asset price, \( \hat{\psi} \), which is greater than \( \bar{\psi}_c \), at which the slope of DE (18) is equal to the slope of DE (20), \( \frac{1}{p} \). In this case, as \( \gamma \) decreases, \( \psi \) rises first, and then declines into \( \frac{v'(1)}{1-p} \) after \( \psi \) passes by \( \hat{\psi} \) as described in Figure 4. On the other hand, if \( \frac{1-c_u}{p} > 1 \), then \( \hat{\psi}_c < \psi < \bar{\psi}_c \) for all \( \gamma \in (v'(1), \hat{\psi}_c) \). Hence, there is no region that \( \psi \) rises as \( \gamma \) decreases in this case.

4 Concentrated Banking System

In this section we characterize the monopolistic equilibrium allocation in which only one banker is legally permitted to issue money by purchasing the asset. The single banker maximizes its monopoly rent, so there must be a social loss from this distortion. On the other hand, this monopolistic market structure has an advantage that the monopoly supplier can adjust the money supply by considering the effect on the prices in general equilibrium. Since the prices can be pinned down by the monopoly supplier in every period, we will focus on the steady states in the following analysis. \(^{17}\)

4.1 Monopolistic Equilibrium

The maximization problem of this monopoly supplier is the same as the problem of the competitive banker in (1)-(3) except that the monopoly supplier can control the aggregate money and asset supplies. That means, the monopoly supplier can choose not only the quantities, \( a_i^t \) and \( \bar{m}_t \), but also the prices, \( q_t \) and \( \psi_t \), and the pledgeability proportion, \( \theta_t \), in the objective function as

\[
\text{Max} \quad \Pi_t = -\psi_t a_i^t + \beta \psi_{t+1} a_i^t + q_t \bar{m}_t - \beta \bar{m}_t. \tag{22}
\]

Thus, the monopoly supplier can internalize the effect of the money issuance on the prices. Remember that in case of default, only the backed assets are seized and the monopoly supplier can keep issuing money by purchasing new asset. Thus, the IC (3) can be relaxed when the profit of the current period increases, but not affected by the future profit. In this respect, the franchise value of the monopoly issuer could be helpful but limited to expand the money supply in (2).

Definition 2. Given \( \gamma \), a monopoly equilibrium consists of the quantities \( (a_t, x_t, m_t) \), the prices \( (\psi_t, q_t) \), and the pledgeability \( \theta_t \), which satisfy equations (2),(5)-(6),(8)-(11), and maximize the monopoly issuer’s profit (22).

\(^{17}\)We can consider the monopoly supplier as one large institution that enables to control the price in the market. For example, the central bank can adjust the short-term interest rate by intervening in the interbank market through Open-market Operations.
4.2 Steady States

The feasibility condition (14) remains in the monopoly equilibrium because it is derived only from the buyer’s first-order condition (9), the binding collateral constraints (2),(5) and the market clearing condition (10). On the other hand, by using (2),(5) and (9)-(10), the objective function (22) can be transformed into

\[-\psi a^i + \beta \psi a^i + q\bar{m} - \beta \bar{m} = -v'(a)(1 - a) + x(u'(x) - 1) = -\frac{x}{\theta}(\frac{1}{\beta} - 1) + x(u'(x) - 1).\]

Thus, the monopoly issuer solves

\[\max_{\bar{x} \geq 0} -\frac{x}{\theta}(\frac{1}{\beta} - 1) + x(u'(x) - 1) \quad (23)\]

subject to the IC (6) and the feasibility condition (14).

(i) Case with \(\theta = 0\): If \(\theta = 0\), then even though money is provided by the monopoly supplier, it will not be accepted in the trade. Thus, the equilibrium allocation is \(x = 0\) and \(a = 1\), which is the same as the one in competitive equilibrium. Also, this equilibrium is supported when \(\gamma \leq v'(1)\).

(ii) Case with \(\theta = 1\): If the IC (6) does not bind, then after plugging \(\theta = 1\) into (23), the first-order condition can be derived as \(u'(x) + xu''(x) = (1 - \sigma_u)u'(x) = \frac{1}{\beta}\), while the feasibility condition (14) holds with \(\theta = 1\). The monopoly equilibrium allocation \((\bar{a}_m, \bar{x}_m)\) is uniquely determined by this first-order condition and the feasibility condition (14): \(\bar{x}_m\) satisfies \((1 - \sigma_u)u'(\bar{x}_m) = \frac{1}{\beta}\) and \(\bar{a}_m\) is pinned down in (14) with \(\bar{x}_m\) and \(\theta = 1\). The positive profit condition, \(\Pi \geq 0\), is also satisfied at \((\bar{a}_m, \bar{x}_m)\) as long as \(\sigma \in (0, 1)\). In order to support this monopoly equilibrium, \(\gamma \geq \bar{\psi}_m := \frac{v'((\bar{a}_m)/\Pi)}{1-\beta}\) is required in (6) for \(\theta = 1\). Thus, when \(\gamma\) is sufficiently large, i.e. \(\gamma \geq \bar{\psi}_m\), the IC (6) does not bind and the equilibrium allocation is determined as \((\bar{a}_m, \bar{x}_m)\).

By comparing with the first-order condition in the unconstrained competitive equilibrium, i.e. \(u'(\bar{x}_c) = \frac{1}{\beta}\), we can find out that \(\bar{x}_m < \bar{x}_c\) because \(\sigma_u < 1\). Therefore, \(\bar{a}_m > \bar{a}_c\) holds from the feasibility condition (14) and \(\bar{q}_m > \bar{q}_c = 1\) also holds from (8). This result is based on the general logic of the monopoly rent maximization. The monopoly issuer reduces the money supply to maximize his/her profit, so the asset consumption increases and the price of money goes up.

(iii) Case with \(\theta \in (0, 1)\): If the IC (6) binds, the feasibility condition (14) is rearranged into (16), and by using (6) and (16), the objective function (23) can be transformed into

\[\max_{x,a \geq 0} xu'(x) - \gamma(1 - a). \quad (24)\]

Therefore, the monopoly issuer solves (24) subject to the feasibility condition (16). Since the objective function (24) will be tangent with the feasibility condition (16) at the monopoly equilibrium.
allocation \((\hat{a}_m, \hat{x}_m)\), we can find it by comparing the slopes of the objective function \((24)\) and the feasibility condition \((16)\) as

\[
- \frac{\partial x}{\partial a} \bigg|_{\Pi_m} = \gamma - \left(1 + \frac{1-a}{a} \sigma_v\right) v'(a) = - \frac{\partial x}{\partial a} \bigg|_{FC_m}.
\]

(25)

Since \(a\) and \(x\) are strictly positively related in \((24)\) while negatively related in \((16)\), the monopoly equilibrium allocation \((\hat{a}_m, \hat{x}_m)\) is uniquely determined from \((24)\) and \((16)\). In order to support this equilibrium allocation \((\hat{a}_m, \hat{x}_m)\) with \(\theta \in (0, 1)\), \(v'(1) < \gamma < \bar{\psi}_m := \frac{v'(\bar{a}_m)}{1-\beta}\) is required from \((6)\).

Remember that the feasibility condition \((16)\) applies for both competitive and monopoly equilibrium. Thus, by comparing the equilibrium condition \((25)\) with the zero-profit condition in the competitive equilibrium \((15)\), we can confirm that \(\hat{x}_m < \hat{x}_c\) and \(\hat{a}_m > \hat{a}_c\) hold with \((\sigma_u, \sigma_v) \in (0, 1) \times (0, 1)\). That means, the monopoly supplier issues less amount of money than the aggregate money supply that the competitive bankers provide. When the IC \((6)\) binds and the pledgeability is determined endogenously, the monopoly supplier prefers to purchase the asset and issue money less, not only because of the monopoly rent, but also because of maintaining the price of the asset as low and raising the pledgeability of the asset to maximize his/her profit, which we analyze in the following subsection.

Finally, note that since \(\hat{a}_m > \hat{a}_c\) holds, the threshold \(\hat{\psi}_m\) is lower than \(\hat{\psi}_c\) as \(\hat{\psi}_m = \frac{v'(\hat{a}_m)}{1-\beta} < \frac{v'(\hat{a}_c)}{1-\beta} = \bar{\psi}_c\). Therefore, we can have a case where the IC \((6)\) binds in competition, but not in monopoly as shown in Figure 5.

### 4.3 Equilibrium Pledgeability

In this subsection we compare the equilibrium pledgeability in competitive and monopoly equilibrium, \(\theta_i\) for \(i \in \{c, m\}\) to understand the main benefit of the market power.

**Lemma 3.** Given \(v'(1) < \gamma < \bar{\psi}_c\), \(\theta_m > \theta_c\) in equilibrium.

**Proof.** See the appendix.

When the IC \((6)\) binds with \(\theta_i \in (0, 1)\) for \(i \in \{c, m\}\), the pledgeability depends on the price of the asset as \(\theta_i = \frac{\gamma}{p\psi_i} - \frac{1}{\beta} + 1\) in a stationary equilibrium. For given \(\gamma\), \(\hat{\psi}_m < \hat{\psi}_c\) holds because \(\hat{a}_m > \hat{a}_c\). Therefore, \(\theta_m > \theta_c\) always holds. More importantly, \(\hat{\psi}_m \theta_m > \hat{\psi}_c \theta_c\) also holds since \(\hat{\psi}_i \theta_i = \frac{\gamma - v'(\hat{a}_i)}{\beta}\). It implies that when the IC \((6)\) binds, the monopoly supplier sets the price of the asset lower than that in competitive equilibrium to raise the pledgeability of the asset, and at that time the pledgeability increases more than the decrease in the price of the asset. Therefore, it is more efficient way to transform one unit of asset into money, because the money supplier can support more money transactions, \(x\), by holding one unit of asset, \(a_i\), in \((13)\). Hence, there is a possibility that the monopoly can improve the outcome of competitive equilibrium.
Additionally, we can address the direction of the equilibrium \( \theta_i \) for \( i \in \{c,m\} \) when the IC (6) binds further with the lower \( \gamma \). Lemma 4 is useful to find out the direction of \( \theta_i \) for \( i \in \{c,m\} \) when \( \gamma \) decreases in each equilibrium.

**Lemma 4.** For each \( i \in \{c,m\} \), given \( v'(1) < \gamma < \bar{\Psi}_i \), \( x_i \) increases in \( \gamma \). If \( \gamma \) is sufficiently small as \( \gamma \leq \frac{v'(1)}{\sigma_u} \), \( a_i \) decreases in \( \gamma \).

**Proof.** See the appendix.

Lemma 4 shows that when the IC (6) binds with \( \theta_i \in (0,1) \) for \( i \in \{c,m\} \), the money trade \( x \) always decreases as \( \gamma \) falls because the pledgeability falls more than the rise of the asset price, \( \psi \). The buyer’s asset holdings, \( a \), may also decrease in \( \gamma \), but must increase when \( \gamma \) is sufficiently low as \( \gamma \leq \hat{\gamma} \), because it is not profitable for the suppliers to purchase the asset and issue money. When \( \gamma \) decreases, \( \theta_c \) decreases and approaches to zero, because

\[
\theta_c = \frac{1-\beta}{\beta} \left( \frac{\gamma}{v'(a_c)} - 1 \right) = \frac{1-\beta}{\beta} \frac{1}{1 - \sigma_u} \frac{1}{u(x_c)} - 1
\]

holds from (15) and \( \frac{dx_c}{d\gamma} > 0 \) holds from Lemma 4. Similarly, \( \theta_m \) also decreases and approaches to zero when \( \gamma \) declines.\(^{18}\) Therefore, we can describe the equilibrium pledgeability \((\theta_c, \theta_m)\) by \( \gamma \) as shown in Figure 5.

\[
\begin{align*}
&\theta_i \\
&1 \\
&O
\end{align*}
\]

\[x_m\] \[\bar{\Psi}_m\] \[\bar{\Psi}_c\] \[\gamma\]

Figure 5: Pledgeability Comparison

5 Welfare Analysis

In this section we describe the efficient allocation in which a planner maximizes the social welfare function given the buyer’s optimization and the market clearing conditions. Notice that the monopoly supplier also considers all the equilibrium conditions as the same as the planner, but he/she maximizes its own profit instead of the social welfare. This welfare analysis allows us to understand the trade-off between the competitive equilibrium and the monopoly equilibrium.

\(^{18}\)We can transform (24) into

\[
\frac{\gamma}{v'(a_m)} = (1 + \frac{1-a_m}{\sigma_v} \frac{1}{v'(a_m)}) \frac{1}{1 - \sigma_u \bar{v}(x_m) - 1}
\]

When \( \gamma \) decreases into \( v'(1) \), \( x_m \) decreases and \( a_m \) increases by Lemma 4. Thus, \( \theta_m \) decreases and will be zero as \( a_m \) approaches to 1.
5.1 Efficient Allocations

By adding the expected utilities across the agents in a stationary equilibrium we can obtain the welfare function,

\[ W = u(x) - x + v(a), \]  

which is shown as \( W \) curve in Figure 6.

**Definition 3.** Given \( \gamma \), an efficient allocation consists of the quantities \( (a_t, x_t, m_t) \), the prices \( (\psi_t, q_t) \), and the pledgeability \( \theta_t \), which satisfy equations (2),(5)-(6),(8)-(11), and (17) with a non-negative profit and maximize the welfare function (26).

If the IC (6) does not bind, the planner maximizes the welfare function (26) subject to the feasibility condition (14) with \( \theta = 1 \).

**Proposition 1.** If \( \gamma \geq \overline{\psi}_c \), then the competitive equilibrium allocation \( (\overline{a}_c, \overline{x}_c) \) is optimal.

**Proof.** See the appendix.

Proposition 1 shows that when the IC (6) does not bind, the competitive equilibrium allocation is optimal. In this model the aggregate liquidity depends on the asset price, given the fixed supply of the asset. As more as money is supplied, the asset consumption \( a \) is reduced, so the price of the asset can go up. Since the pledgeability remains at \( \theta = 1 \), the more money transactions can be supported as the price rises. Therefore, the welfare can be maximized, although the decentralized bankers issue money until the profit becomes zero. Consequently, when the IC does not bind, the monopoly equilibrium is sub-optimal because the money supply is reduced as \( \bar{x}_m < \bar{x}_c \) and \( \bar{a}_m > \bar{a}_c \) hold, given the same feasibility condition.

![Figure 6: Constrained Equilibrium Allocations](image-url)
On the other hand, if the IC (6) binds, the planner maximizes the welfare function (26) subject to the feasibility condition (16). Since the feasibility condition (16) is hump-shaped as $FC_{\theta \in (0,1)}$ curve in Figure 6 and the welfare function (26) increases in both $a$ and $x$, we focus on the right-hand side of $FC_{\theta \in (0,1)}$ curve, where $a$ and $x$ are negatively related, to find out the constrained optimal allocation.

The slopes of the welfare function and the feasibility condition can be equal as

$$\frac{\partial x}{\partial a} \bigg|_{W} = \frac{v'(a)}{u'(x) - 1} = \gamma - (1 + \frac{1 - a}{a} \sigma_v) v'(a) = -\frac{\partial x}{\partial a} \bigg|_{FC_{\theta \in (0,1)}} \tag{27}$$

at the constrained optimal allocation $(\hat{a}_p, \hat{x}_p)$. Since $a$ and $x$ are positively related in (27), the constrained optimal allocation $(\hat{a}_p, \hat{x}_p)$ is uniquely determined by (16) and (27).

**Proposition 2.** When $v'(1) < \gamma < \bar{\psi}_c, \hat{a}_p > \hat{a}_c$. Similarly, when $v'(1) < \gamma < \bar{\psi}_m, \hat{a}_p < \hat{a}_m$.

**Proof.** See the appendix.

Proposition 2 shows that when the IC (6) binds with $\theta \in (0,1)$, the decentralized bankers provide more money than the optimal level in the competitive equilibrium, while the monopoly supplier still provides the money supply less than the optimal level. In the case of competitive setting, if the IC (6) binds, issuing money requires an additional quantity of the asset to secure the money transaction, thus the price of the asset rises due to the higher demand. Then, it becomes more likely to fake the collateral, so the pledgeability, $\theta$, decreases and the IC (6) will bind further. In this respect, the individual bankers should not provide money excessively for social optimality, but they provide money until there is no profit. On the other hand, the monopoly supplier takes all the equilibrium conditions into account, but he/she maximizes his/her own profit. Proposition 2 confirms that the money supply can be also reduced excessively in the constrained monopoly equilibrium because of the distortion associated with the monopoly rent. Since both constrained competitive and monopoly equilibria are sub-optimal, in the next subsection we address in what circumstance the monopolistic market structure can be better off than competition.

### 5.2 Welfare Comparison

In this subsection we will compare welfare of competitive equilibrium with that of monopoly equilibrium when the IC (6) binds to understand the trade-off between competition and monopoly for the money issuance.

**Proposition 3.** If $\sigma_u$ is sufficiently small and $\sigma_v$ is sufficiently large, then welfare of constrained monopoly equilibrium is greater than welfare of constrained competitive equilibrium, $W_M > W_C$, when $\gamma = \bar{\psi}_m$.

19Wallace (1983) points out that a decreasing cost of inhibiting counterfeit can give us a reason for a single supplier because of externality. However, the externality from the limited commitment in this paper does not depend on the feature of the specific cost function as shown in his paper.
Proposition 3 shows that there exists a set of parameters in which the monopoly equilibrium allocation dominates the competitive equilibrium allocation. When $\gamma$ approaches to $\bar{\psi}_m$, the IC (6) holds in competitive equilibrium, i.e. $\theta_c \in (0,1)$ while the IC (6) does not hold yet in monopoly equilibrium, i.e. $\theta_m = 1$. Therefore, the gap between the pledgeable proportions of these equilibria becomes larger as shown in Figure 5. In this case we can show that the unconstrained monetary equilibrium $(\hat{a}_m, \hat{x}_m)$ at point D is better off than the constrained optimal allocation $(\hat{a}_p, \hat{x}_p)$ at point P in Figure 6, since the constrained optimal allocation $(\hat{a}_p, \hat{x}_p)$ always dominates the constrained competitive equilibrium $(\hat{a}_c, \hat{x}_c)$. If $\sigma_u$ is sufficiently small, then the unconstrained monetary equilibrium $(\hat{a}_m, \hat{x}_m)$ becomes closer to the unconstrained competitive equilibrium $(\hat{a}_c, \hat{x}_c)$. On the other hand, if $\sigma_v$ is sufficiently large, then the slope of the feasibility curve, $FC_{\theta\in(0,1)}$ become more generous because the price of the asset becomes more elastic to the changes in the asset demand, and thus the pledgeability is more likely to fall further.\footnote{The price elasticity of demand for the asset decreases in $\sigma_v$.} Therefore, the Eq. (27) for the constrained optimal allocation $(\hat{a}_p, \hat{x}_p)$ shifts downward and so does the constrained optimal allocation. Hence, we can show that for given $\gamma = \bar{\psi}_m$, the monopoly provides higher welfare than the competition.

The directions of the parameters $(\sigma_u, \sigma_v)$ are reasonable when we consider the trade-off between the monopoly and the competition. As $\sigma_u$ falls, the distortion generated by the monopoly rent becomes smaller as discussed in subsection 4.2. Additionally, if the price of the asset is more elastic to the change in the demand with the higher $\sigma_v$, the amplification effect becomes greater since the price goes up further given the same shortage in supply, so the inefficiency in the competition would increase.

6 Regulations

As we learned in the previous section the key solution is to raise the level of the pledgeability, $\theta$, in the IC (6), although it is endogenously determined in general equilibrium. In this section I consider two possible regulations, reserve requirements and an entry cost, to improve the competitive equilibrium allocation.

6.1 Reserve Requirements

Suppose that $\delta$ proportion of the asset holdings of the bankers must be kept in the central bank. Then, only $1 - \delta$ proportion of their asset holdings is exposed to the collateral misrepresentation. While the buyer’s problem remains, the collateral constraint (2) and the IC (3) in the banker’s

\[ \text{Proof. See the appendix.} \]
problem are changed into
\[ \beta \psi_{t+1} a_t (1 - \delta) \theta_t + \beta \psi_{t+1} a_t \delta \geq \beta \bar{m}_t, \tag{28} \]
and
\[ -\gamma a_t (1 - \delta) - \psi_t a_t \theta + q_t \bar{m}_t \leq -\psi_t a_t + \beta \psi_{t+1} a_t + q_t \bar{m}_t - \beta \bar{m}_t, \tag{29} \]
respectively. By plugging (28) into the IC (29), we can show that the pledgeability equation remains the same as (6). Then, at the steady state, the zero-profit condition (12) and the feasibility condition (14) can be modified with the reserve requirement \( \delta \) as
\[ \{ \delta + \theta (1 - \delta) \} \{ u'(x) - 1 \} = \frac{1}{\beta} - 1, \tag{30} \]
and
\[ x = \beta \psi \{ \delta + \theta (1 - \delta) \} (1 - a) = \frac{\beta \psi}{1 - \beta} \{ \delta + \theta (1 - \delta) \} (1 - a). \tag{31} \]

If the IC (6) does not bind with \( \theta = 1 \), then these equilibrium conditions (30)-(31) are the same as (12) and (14) with \( \theta = 1 \), so the equilibrium allocation \( (\bar{a}_c, \bar{x}_c) \) is maintained. However, if the IC (6) binds, then the inefficiency generated by the reduced pledgeability can be mitigated since \( \delta + \theta (1 - \delta) \) is applied for both conditions (30)-(31) instead of \( \theta \). More importantly, by using the IC (6) the feasibility condition (31) can be rearranged into
\[ x = \{(1 - \delta) \gamma + (\frac{\delta}{1 - \beta} - 1) v'(a)\} (1 - a). \tag{32} \]

Notice that if \( \delta \geq 1 - \beta \), this feasibility condition (32) is no more hump-shaped, and \( a \) and \( x \) are now negatively related. That means, when the competitive issuers purchase the asset to issue money, the price of the asset would rise, but the pledgeability does not decrease much, so the liquidity will not be dried-up as before. In this case the equilibrium allocation \( (\bar{a}_c, \bar{x}_c) \) is uniquely determined because \( a \) and \( x \) are still positively related in the zero-profit condition (30) with (6).

**Corollary 1.** If the reserve requirement is imposed with \( \delta \geq 1 - \beta \), then the competitive equilibrium allocation \( (\bar{a}_c, \bar{x}_c) \) is constrained optimal when \( \gamma < \bar{\psi}_c \).

**Proof.** See the appendix.

Corollary 1 shows that the competitive equilibrium allocation can be restored as optimal just by imposing a low level of reserve requirement, \( \delta = 1 - \beta \). The cost for holding one unit of genuine asset, \( \psi (1 - \beta) \), depends on the asset price, \( \psi \). Thus, if the asset price goes up, then it becomes more attractive to produce a fake asset at \( \gamma \), so the pledgeability \( \theta \) can fall sharply. Without a reserve requirement, \( \delta = 0 \), when the bankers purchase the asset to issue money, the asset price, \( \psi \), rises, but the pledgeability, \( \theta \), decreases more than that. Consequently, the asset price multiplied by the
pledgeability term, \( \psi \theta \), decreases as the banker purchase more assets. However, if the bankers are required to hold \( \delta \) proportion of the genuine asset, then \( \psi \theta \) can rather increase when they buy the asset and issue money, and so the feasibility condition (32) will be no more hump-shaped. Since the bankers hold \( \delta \) proportion of the genuine asset, when the price of the asset increases, the bankers earn a proportional benefit from their genuine asset-holdings, \( \delta \psi \), thus the rise in the asset price may not induce the pledgeability to fall further.

Finally, note that even though \( \theta = 0 \) at \( \gamma \leq v'(1) \), the money trade will not be shut down here because \( \bar{x}_c > 0 \) holds as long as \( \delta > 0 \). Although the cost for imposing reserve requirements is not explicitly considered in this model, it must be valuable to enforce a minimum reserve requirement when the money is not supplied any more in an economy.

The modified zero-profit condition (30) and the feasibility condition (31) with a reserve requirement, \( \delta > 0 \), are described in Figure 7. Since the zero-profit condition (30) rotates counterclockwise and the bent feasibility condition (31) is straightened, the equilibrium allocation with the reserve requirement at point E always dominates the equilibrium allocation without the reserve requirement at point C.

![Figure 7: Reserve Requirements](image)

### 6.2 Entry Cost

Now suppose that the government can collect an entry cost from the bankers. In each period \( CM \) the bankers require to pay \( \kappa \) units of \( CM \) good to continue their business. In this case the banker’s problem (1)-(3) remains, but an entry condition will be added as

\[
\Pi_t := -\psi_t a_t^i + \beta \psi_{t+1} a_t^i + q_t \bar{m}_t - \beta \bar{m}_t \geq \kappa.
\]  (33)
If the IC (6) does not bind, the equilibrium conditions (12) and (14) are maintained at the steady state. By plugging (2),(5),(8)-(11) with $\theta = 1$ into (33), we can obtain
\[
\Pi = xu'(x) - \frac{1}{\beta} x \geq \kappa.
\]
Therefore, when we define $\hat{x}$ as $\hat{x}u'(\hat{x}) - \frac{1}{\beta} \hat{x} = \kappa$, $\hat{x} < \hat{x}_c$ holds because $\Pi = \hat{x}_c u'(\hat{x}_c) - \frac{1}{\beta} \hat{x}_c = 0$ holds in the unconstrained competitive equilibrium.\(^{21}\) In this respect, the money supply is restricted further and the welfare also decreases by imposing an entry cost, if the IC (6) does not bind.

When the IC (6) binds with $\theta \in (0, 1)$, the equilibrium conditions (15)-(16) remain as the same. However, by using (6) we can rearrange the entry condition (33) into
\[
\Pi = xu'(x) - \gamma(1 - a) \geq \kappa,
\]
which is described as $\Pi = \kappa$ curve in Figure 8. Note that the line AC on $FC_{\delta=0}$ curve is only feasible with $\Pi = \kappa$, and the bankers will provide money until their profit is equal to the required one, $\kappa$, at point C. Therefore, if the entry cost, $\kappa$, can be set as the profit at the constrained optimal allocation at point P, the welfare can be improved.

![Figure 8: Entry Cost](image)

**Corollary 2.** When $v'(1) < \gamma < \hat{\psi}_c$, the optimal entry cost is $\kappa^* = \hat{x}_p u'(\hat{x}_p) - \gamma(1 - \hat{a}_p) > 0$.

\(^{21}\)In fact, the profit function, $\Pi = xu'(x) - \frac{1}{\beta} x$, is hump-shaped, there are two solutions that satisfies with $\Pi = \kappa > 0$. Since the bankers provide the money until the profit goes to zero, the one with the larger $x$, i.e. $\hat{x}$, is the equilibrium allocation.
Proof. See the appendix.

As shown in Corollary 2, a strictly positive entry cost can improve welfare because the required profit can inhibit the bankers to provide money excessively. However, comparing to the reserve requirements, the effect of the entry barrier is limited, because it can neither expand the feasibility set, nor eliminate the non-monetary equilibrium at $γ \leq v'(1)$.

7 Unsecured debt

In this section I extend the model by incorporating unsecured debt in order to know whether our main result still holds even when the banks can choose their money’s convertibility voluntarily in the model. I assume that if a banker refuses to redeem the money in the CM, then the banker cannot issue money any more. With this modified assumption, an individual banker solves the following problem in the CM of period $t$:

$$J_t = \max_{a_t, \bar{m}_t \geq 0} a_t^i + \beta \psi_{t+1} a_t^i + q_t \bar{m}_t - \beta \bar{m}_t + \beta J_{t+1}$$

subject to a collateral constraint,

$$\beta \psi_{t+1} a_t^i \theta_t + \beta J_{t+1} \geq \beta \bar{m}_t$$

and an incentive constraint,

$$-γ a_t^i + q_t \bar{m}_t \leq -\psi_t a_t^i + \beta \psi_{t+1} a_t^i + q_t \bar{m}_t - \beta \bar{m}_t + \beta J_{t+1}.$$  \hspace{1cm} (36)

Note that the money transactions are backed not only by the asset holdings, but also by the continuation value of the bankers.

In a competitive equilibrium an individual bank’s profit must be zero, $Π_t = -\psi_t a_t^i + \beta \psi_{t+1} a_t^i + q_t \bar{m}_t - \beta \bar{m}_t = 0$, in each period, because money can be issued at any time by purchasing the asset. If $Π_t > 0$, then the bankers will buy more asset and issue money until the profit becomes zero.\footnote{This point is different from Sanches (2016): In his paper, the profit must be strictly positive to support the money transactions without real assets. However, in my paper, even though the continuation value is zero, $J_{t+1} = 0$, money can be issued by holding the asset.} Therefore, in the steady state, $J = \frac{Π_1}{1-β} = 0$ holds, so the competitive equilibrium allocation in the previous section is maintained although we allow the bankers to use their reputations.

In case of monopoly, on the contrary, the single supplier does not hold the asset if unsecured credit is available. In a stationary equilibrium, by using the equilibrium conditions, we can trans-
form the maximization problem (34)-(35) into

\[
\begin{align*}
\max_{a, x \geq 0} & \quad -v'(a)(1 - a) + x\{u'(x) - 1\} \\
\text{subject to} & \quad \beta\psi(1 - a)\theta + \frac{\beta \Pi}{1 - \beta} = -\frac{\beta v'(a)(1 - a)}{1 - \theta} + \frac{\beta x\{u'(x) - 1\}}{1 - \beta} \geq x,
\end{align*}
\]

where \( J = \frac{\Pi}{1 - \beta} \) and \( \psi = \frac{v'(a)}{1 - \beta} \). Notice that the monopoly banker will not hold the asset to issue money, i.e. \( a = 1 \), in equilibrium regardless of the level of \( \theta \). Holding the asset is costly in the objective function (37) and cannot relax the collateral constraint (38), because it also requires to hold the asset in the future and that reduces the continuation value of the banker. Therefore, the maximization problem (37)-(38) can be simplified into

\[
\begin{align*}
\max_{x \geq 0} & \quad x\{u'(x) - 1\} \\
\text{subject to} & \quad \frac{\beta x\{u'(x) - 1\}}{1 - \beta} \geq x.
\end{align*}
\]

If \( \beta \) is sufficiently high, then the debt constraint (40) does not bind. Thus, the money trade will be \( x = \bar{x} \) where \( (1 - \sigma_u)u'(\bar{x}) = 1 \). However, if the debt constraint (40) binds then the money transactions are restricted as \( x = \hat{x} \) where \( \beta(1 - \sigma_u)u(\hat{x}) = \hat{x} \). There is no concern for collateral misrepresentation, since the monopoly supplier does not hold the asset. However, if \( \sigma_u \) is too large, then it could be worse than competition because the monopoly supplier will not issue money sufficiently to maximize its own profit.

We can compare this monopoly equilibrium allocation \((a_m, x_m)\) with the unconstrained competitive equilibrium allocation \((\bar{a}_c, \bar{x}_c)\). If the agents are patient and \( \sigma_u \) is sufficiently small, then the debt constraint (40) does not bind and the demand for money is more elastic to the price, i.e. \( 1 - \sigma_u > \beta \), thus the monopoly allocation always dominates the competitive outcome regardless of \( \gamma \) because \( a_m = 1 \) and \( x_m = \hat{x} > \bar{x}_c > 0 \) where \( \beta u'(\bar{x}_c) = 1 \). On the other hand, if \( \sigma_u \) approaches to 1, the money trade could decrease into zero in the monopoly case, so there will be a trade-off because the monopoly supplier would reduce the money supply excessively even though he/she does not hold the asset.

Exceptionally, in case of \( \gamma \leq v'(1) \) there arises a non-monetary equilibrium in competition, so the monopoly with \( a_m = 1 \) and \( x_m > 0 \) is always better off. However, in case of \( \gamma > v'(1) \) there exists a trade-off between the distortion from the monopoly rent and the cost for holding the asset plus the inefficiency from collateral misrepresentation in competition.

\[23\]Note that this monopoly equilibrium with unsecured debt is the same as the one in Sanches (2016), because the monopoly banker does not hold the asset in equilibrium.
8 Conclusion

In this paper we study the role of market power in providing asset-backed money under asymmetric information. If it is hard to verify the quality of bank assets, the competitive equilibrium allocation can be sub-optimal because the individual bankers cannot internalize the effect of issuing money on the prices and the haircut. When the counterfeiting cost is sufficiently low, the supply of liquidity can be reduced excessively as the haircut increases. Once the aggregate supply becomes scarce, the price of the backed asset rises in the market, then the haircut will go up further because the moral hazard incentive is encouraged. Therefore, the liquidity in an economy can be dried up through this amplification mechanism. We show that introducing a monopolistic market structure can improve welfare by internalizing the aggregate supply effect on prices and raising transparency with a positive profit. In this respect an entry barrier on the financial intermediaries can be justified to reduce their moral hazard incentives with strictly positive profits. Moreover, we also show that reserve requirements are effective to restore the constrained efficiency in competitive equilibrium when the liquidity dry-up occurs.

This paper takes one step to understand the role of market structure for the liquidity creation of the financial institutions. However, there are many unexplored questions and extensions related to this topic. For example, what would be the appropriate assets for the central bank under asymmetric information? If the asset portfolio of the financial intermediaries can be verified at a cost, then what would happen? I hope that we can explore these remaining issues in near future.

References


Appendix

A  Proofs

Lemma 1. When \( \theta \in (0,1) \), the stationary monetary equilibrium allocation \((\hat{a}_c, \hat{x}_c)\) is uniquely determined by (15)-(16).

Proof. By plugging (15) into (16), we can modify (16) into \( xu'(x) = \gamma (1 - a) \) as long as \( x > 0 \). Since \( xu'(x) \) increases in \( x \) with \( \sigma_u < 1 \), \( x \) and \( a \) are negatively related in this modified equilibrium condition. When \( a \) decreases into \( \hat{a} \) where \( \gamma = \nu'(\hat{a}) \), \( x \) approaches to zero in (15) whereas \( x > 0 \) in the modified condition (16). When \( a \) approaches to 1, \( x = 0 \) in the modified condition (16), while \( x > 0 \) in (15). Thus, there exists a unique equilibrium allocation \( \hat{a}_c \) in \((\hat{a},1)\), which is associated with \( \hat{x}_c > 0 \). QED

Lemma 2. Given \( \gamma \), the following hold:

(a) If \( \gamma \leq \nu'(1) \), there exists only a non-monetary stationary equilibrium with \( x = 0 \) and \( a = 1 \).

(b) If \( \nu'(1) < \gamma < \hat{\psi}_c \), there is a unique steady state with \((\hat{a}_c, \hat{x}_c)\).

(c) If \( \gamma \geq \hat{\psi}_c \), there is a unique steady state with \((\hat{a}_c, \hat{x}_c)\).

Proof. Since it is already shown that the equilibrium allocation in each case (a)-(c) is supported in the corresponding region of \( \gamma \), here I will discuss that the allocation is unique in each case.

(a) When \( \gamma \leq \nu'(1) \), \( \theta > 0 \) is not feasible. For given \( \theta > 0 \), we will obtain the price as \( \hat{\psi}_c = \frac{\nu'(\hat{a}_c)}{1-\beta} \) or \( \hat{\psi}_c = \frac{\nu'(\hat{a}_c)}{1-\beta} \) in equilibrium. However, in either cases \( \theta = \min[1, \max\{\frac{1-\psi+\beta\psi}{\beta\psi}, 0\}] \) must be zero since \( \gamma \leq \nu'(1) \leq \min\{\nu'(\hat{a}_c), \nu'(\hat{a}_c)\} \). Therefore, the non-monetary equilibrium with \( \theta = 0 \) is unique at \( \gamma \leq \nu'(1) \).

(b) Similarly, when \( \nu'(1) < \gamma < \hat{\psi}_c \), either \( \theta = 0 \) or \( \theta = 1 \) are not feasible. If \( \theta = 0 \) holds in equilibrium, then \( a = 1 \), so \( \psi = \frac{\nu'(1)}{1-\beta} \). Then, \( \gamma \leq (1-\beta)\psi = \nu'(1) \) is required for \( \theta = 0 \), but it contradicts \( \nu'(1) < \gamma \). On the other hand, if \( \theta = 1 \) holds in equilibrium, then \( \gamma \geq \hat{\psi}_c \) is required, but it also contract \( \gamma < \hat{\psi}_c \).

(c) When \( \gamma \geq \hat{\psi}_c \), \( \theta < 1 \) is not feasible. If \( \theta = 0 \) holds in equilibrium, then \( a = 1 \), so \( \psi = \frac{\nu'(1)}{1-\beta} \). Then, \( \gamma \geq \hat{\psi}_c > \frac{\nu'(1)}{1-\beta} = \psi \) contradicts \( \theta = 0 \). Finally, if \( \theta \in (0,1) \) holds in equilibrium, then there must be the equilibrium price \( \hat{\psi}_c \) such that \( \hat{\psi}_c > \gamma \geq \hat{\psi}_c \) holds. However, we can show that it contradicts by using dynamic equations (18) and (20). Note that (18) is strictly convex and pass the 45 degree line at \( \hat{\psi}_c \) as shown in the right panel of Figure 3. Thus, as long as the slope of (20) is greater than 1 and \( \gamma \geq \hat{\psi}_c \), there exists no more intersection for \( \psi_t > \gamma \). QED

Lemma 3. Given \( \nu'(1) < \gamma < \hat{\psi}_c \), \( \theta_m > \theta_c \) in equilibrium.
By the Implicit function theorem, we can have \( \partial \) while the feasibility condition (16) shifts downward. To find out the direction of \( \hat{\gamma} \) numerator is also strictly positive, then \( \partial \). Since \( a \) and \( x \) are negatively related in (14) with

**Proof.**

**Proposition 1.**

When \( v'(1) < \gamma < \tilde{\psi}, x_i \) increases in \( \gamma \). If \( \gamma \) is sufficiently small as \( \gamma \leq \frac{v'(1)}{c''}, a_i \) decreases in \( \gamma \).

**Proof.**

If \( \gamma \) is reduced, \( \hat{x}_m \) decreases because the first-order condition (15) shifts to the right while the feasibility condition (16) shift downward. To find out the direction of \( \hat{a}_c \), we transform (16) into \( \hat{x}_c = \frac{(1 - \hat{a}_c)(1 - \hat{a}_c)}{\gamma} \) by assuming \( u(x) = \frac{1-a}{1-a} \). Since \( \hat{x}_c = (1 - \hat{a}_c)(\gamma - v'(\hat{a}_c)) \) holds from (15)-(16), we can have \( (1 - \hat{a}_c)(1 - \hat{a}_c) = (1 - \hat{a}_c)(\gamma - v'(\hat{a}_c)) \). By the Implicit function theorem, we can address \( \frac{\partial a}{\partial \gamma} = \frac{-1}{(1-a)(\gamma-v'(\hat{a}_c))} \). Since the denominator is strictly positive, if the numerator is also strictly positive, then \( \frac{\partial a}{\partial \gamma} < 0 \) will hold. By using (15), the numerator can be rearranged into \( 1 - a \sigma_u > \{(1 - \hat{a}_c)(\gamma - v'(\hat{a}_c))\}^u = \frac{1}{\gamma u(x)} = 1 - \frac{v'(\hat{a}_c)}{\gamma} \). Thus, if \( a \sigma_u \gamma > v'(\hat{a}_c) \), then \( \frac{\partial a}{\partial \gamma} < 0 \), whereas if \( a \sigma_u \gamma < v'(\hat{a}_c) \), \( \frac{\partial a}{\partial \gamma} > 0 \). Since \( v'(1) < v'(\hat{a}_c) \), if \( a \sigma_u \gamma < v'(1) \), \( \frac{\partial a}{\partial \gamma} < 0 \).

Similarly, when \( \gamma \) is reduced, \( \hat{x}_m \) decreases because the first-order condition (24) shifts to the right while the feasibility condition (16) shifts downward. To find out the direction of \( \hat{a}_m \), we transform (24) into \( (x_m)^{u(1)} = 1 - a \sigma_u \{(1 - \hat{a}_m)(\gamma - v'(\hat{a}_m))\}^u \) with the assumption \( u(x) = \frac{1-a}{1-a} \). Since \( (x_m, \hat{a}_m) \) satisfy with (16), we can have \( \{(1 - \hat{a}_m)(\gamma - v'(\hat{a}_m))\}^u \) where \( k(\hat{a}_m) := 1 + \frac{1-a}{\hat{a}_m} \sigma_u \).

By the Implicit function theorem, we can have \( \frac{\partial a}{\partial \gamma} = \frac{-1}{(1-a)(\gamma-v'(\hat{a}_m))} \). Note that \( \gamma - v'(\hat{a}_m) > 0 \) because \( \hat{a}_m > \hat{a} \). Since the denominator is strictly negative, if the numerator is also strictly negative, then \( \frac{\partial a}{\partial \gamma} < 0 \) will hold. By using (24), the numerator is strictly negative when \( 1 - \frac{a \sigma_u}{1-a} > \{(1 - \hat{a}_m)(\gamma - v'(\hat{a}_m))\}^u = \frac{1}{\gamma u(x)} = 1 - \frac{k(\hat{a}_m)v'(\hat{a}_m)}{\gamma} \). Given \( a \sigma_u \) and \( \gamma \leq \frac{v'(1)}{c''} \), then \( a \sigma_u \gamma < v'(1) \), so \( v'(\hat{a}_m) > 1 + \sigma_u > \frac{v'(\hat{a}_m)}{\gamma} \) holds because of \( k(\hat{a}_m) > 1 \) and \( v'(\hat{a}_m) < \gamma \). Then, the numerator is strictly negative and \( \frac{\partial a}{\partial \gamma} < 0 \). QED

**Proposition 1.**

If \( \gamma \geq \tilde{\psi} \), then the competitive equilibrium allocation \( (\hat{a}_c, \hat{x}_c) \) is optimal.

**Proof.**

Since \( a \) and \( x \) are negatively related in (14) with \( \theta = 1 \), we can compare the slopes of the welfare function (26) and the feasibility condition (14) with \( \theta = 1 \) at the allocation \( (\hat{a}_c, \hat{x}_c) \) as

\[
-\frac{\partial x}{\partial a} \bigg|_{W} = \frac{\partial v'(\hat{a}_c)}{\partial (\hat{x}_c)} - 1 = \gamma - v'(\hat{a}_c) < \gamma - (1 - \sigma_u a \sigma_u) v'(\hat{a}_c) = -\frac{\partial x}{\partial a} \bigg|_{FC}.
\]

Since the slope of (14) is greater than the slope of (26), the welfare can improve as the money supply increases further. However, the allocation with the larger \( x \) cannot be supported as an equilibrium because of the strictly negative profit. Thus, the allocation \( (\hat{a}_c, \hat{x}_c) \) is optimal. QED

**Proposition 2.**

When \( v'(1) < \gamma < \tilde{\psi}, \hat{a}^p > \hat{a}^c \). Similarly, when \( v'(1) < \gamma < \tilde{\psi}, \hat{a}^p < \hat{a}^m \).
Proof. We can compare the slopes of the welfare function (26) and the feasibility function (16) at \((\hat{a}_C, \hat{x}_C)\). Since \((\hat{a}_C, \hat{x}_C)\) satisfies (15), we can rewrite the slope of the welfare function as \(\frac{\partial \bar{a}}{\partial \hat{a}} \mid_W = \frac{\bar{v}^\prime(\hat{a}_C)}{u(\hat{x}_C) - 1}\). The slope of the feasibility condition at \((\hat{a}_C, \hat{x}_C)\) is \(\frac{\partial \bar{a}}{\partial \hat{a}} \mid_{FC} = \gamma - (1 + \frac{\hat{a}_C}{\hat{x}_C})v^\prime(\hat{a}_C)\) from (16). Since \(\frac{\hat{a}_C}{\hat{x}_C} \sigma_v > 0\), \(\frac{\partial \bar{a}}{\partial \hat{a}} \mid_W > \frac{\partial \bar{a}}{\partial \hat{a}} \mid_{FC}\) at \((\hat{a}_C, \hat{x}_C)\). Therefore, \(\hat{a}_p > \hat{a}_C\) as shown in Figure 6.

Similarly, we can compare the slopes of the welfare function (26) and the feasibility function (16) at \((\hat{a}_M, \hat{x}_M)\). Since \((\hat{a}_M, \hat{x}_M)\) satisfies (24), we can rewrite the slope of the welfare function as \(\frac{\partial \bar{a}}{\partial \hat{a}} \mid_W = \frac{\bar{v}^\prime(\hat{a}_M)}{u(\hat{x}_M) - 1}\), by plugging (24) into \(\frac{\partial \bar{a}}{\partial \hat{a}} \mid_W = \frac{\bar{v}^\prime(\hat{a}_M)}{u(\hat{x}_M) - 1}\). The slope of the feasibility condition at \((\hat{a}_M, \hat{x}_M)\) is \(\frac{\partial \bar{a}}{\partial \hat{a}} \mid_{FC} = \gamma - (1 + \frac{\hat{a}_M}{\hat{x}_M})v^\prime(\hat{a}_M) = \frac{w(\hat{a}_M)}{u(\hat{x}_M) - 1}\) by (24). Since \(\frac{\hat{a}_M}{\hat{x}_M} \sigma_v > 0\) and \(1 - \sigma_u v^\prime(\hat{x}_M) > 1\) is required for the positive value of the slope, \(\frac{\partial \bar{a}}{\partial \hat{a}} \mid_W < \frac{\partial \bar{a}}{\partial \hat{a}} \mid_{FC}\) at \((\hat{a}_M, \hat{x}_M)\). Therefore, \(\hat{a}_p < \hat{a}_M\) and \(\hat{x}_p > \hat{x}_M\) as shown in Figure 6. QED

Proposition 3. If \(\sigma_u\) is sufficiently small and \(\sigma_v\) is sufficiently large, then welfare of constrained monopoly equilibrium is greater than welfare of constrained competitive equilibrium, \(W_M > W_C\), when \(\gamma = \hat{\psi}_M\).

Proof. When \(\gamma = \hat{\psi}_M\), the IC (6) does not bind in monopoly while it binds in competition. Therefore, the equilibrium allocation in monopoly is \((\hat{a}_M, \hat{x}_M)\) at point D while the equilibrium allocation in competition is \((\hat{a}_C, \hat{x}_C)\) at point C as described in Figure 6. Since the welfare of constrained optimal allocation \((\hat{a}_P, \hat{x}_P)\) at point P is greater than one at point C, I will show that welfare at point D is greater than welfare at point P when \(\gamma = \hat{\psi}_M\). Note that a and x are positively related in (27) and (27) passes point P. Thus, it is sufficient to show that point D is located above the intersection of (27) and (14) with \(\theta = 1\) as described in Figure 6. Let’s denote \(\bar{a}\) as the the asset-holdings, a, when \(x = \bar{x}_M\) holds in (27). Then, \(\bar{a}\) satisfies with (27) as

\[
\frac{v^\prime(\bar{a})}{u(\bar{x}_M) - 1} = \gamma - (1 - \sigma_v + \frac{\sigma_u}{\bar{a}})v^\prime(\bar{a}).
\]

By plugging \(\beta(1 - \sigma_u)u^\prime(x_M) = 1\) and \(\gamma = \hat{\psi}_M\), we can rearrange this equation as

\[
\frac{v^\prime(\hat{a}_M)}{v^\prime(\bar{a})} = (1 - \beta)\{\frac{\beta(1 - \sigma_u)}{1 - \beta(1 - \sigma_u)} + (1 + \frac{1 - \bar{a}}{\bar{a}})\sigma_v\}.
\]

When \(\sigma_u\) and \(\sigma_v\) approach to zero and one, respectively, the right-hand side becomes \(\frac{1}{n} \geq 1\), so \(\bar{a} = \bar{a}_M\) can hold with sufficiently small \(\sigma_u > 0\) and sufficiently large \(\sigma_v < 1\) as long as \(\bar{a} < 1\). QED

Corollary 1. If the reserve requirement is imposed with \(\delta \geq 1 - \beta\), then the competitive equilibrium allocation \((\hat{a}_C, \hat{x}_C)\) is constrained optimal when \(\gamma < \hat{\psi}_C\).

Proof. Since a and x are negatively related in (32) with \(\delta \geq 1 - \beta\), we can compare the slopes of the welfare
function (26) and the feasibility condition (32) at the allocation $(\tilde{a}_c, \tilde{x}_c)$ as

$$-\frac{\partial x}{\partial a} \bigg|_{W} = \frac{v'(\tilde{a}_c)}{u'(\tilde{x}_c) - 1} = \gamma(1-\delta) + \left(\frac{\delta}{1-\beta} - 1\right)v'(\tilde{a}_c)$$

$$< \gamma(1-\delta) + \left(\frac{\delta}{1-\beta} - 1\right)(1 - \sigma_v + \frac{\sigma_v}{\tilde{a}_c})v'(\tilde{a}_c) = -\frac{\partial x}{\partial a} \bigg|_{FC}.$$ 

Since the slope of (32) is greater than the slope of (26), the welfare can improve as the money supply increases further. However, the allocation with the larger $x$ cannot be supported as an equilibrium because of the strictly negative profit. Thus, the allocation $(\tilde{a}_c, \tilde{x}_c)$ is optimal. QED

**Corollary 2.** When $v'(1) < \gamma < \bar{\psi}_c$, the optimal entry cost is $\kappa^* = \hat{x}_p u'(\hat{x}_p) - \gamma(1 - \hat{a}_p) > 0$.

**Proof.** When the IC (6) binds, the feasibility condition (16) is not affected by the entry cost. Thus, the welfare is still maximized at point $P$ in Figure 8. Since the profit of the bankers increases as the allocation moves from point $C$ to $P$, the equilibrium allocation can be $(\hat{a}_p, \hat{x}_p)$ when the entry cost is equal to the profit at point $P$ as $\kappa^* = \hat{x}_p u'(\hat{x}_p) - \gamma(1 - \hat{a}_p)$. QED