Large Excess Reserves, Central Bank Digital Currency and Monetary Policy*

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Abstract

In this paper I examine the effect of introducing an account-based central bank digital currency(hereafter CBDC) on liquidity insurance and monetary policy implementation. An asset-exchange model is constructed with idiosyncratic liquidity risk, in which one type of agents require currency and/or CBDC to consume while the other type of agents can use any assets to trade. There arises a liquidity insurance to distribute assets efficiently by type. Since central bank reserve accounts are accessible by the public directly, the large excess reserves(hereafter LER) in a floor system can make it difficult to separate the types under private information. Therefore, raising the interest on reserves in the floor system can reduce the aggregate liquidity excessively, and the equilibrium allocation with the LER can be sub-optimal.

Key Words: Interest on Reserves, Floor System, Liquidity Insurance, Truth-telling Constraints, Liquidity Trap, Aggregate Liquidity

JEL Codes: E42, E44, E52.

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1 Introduction

Recent development in blockchain technology have led not only a number of private digital currencies such as Bitcoin and Ethereum to be widely spread in real world, but also the central bankers to be interested in issuing its own digital currency to the public. Technically, central banks have already issued account-based digital currency in a form of central bank reserves.[1] However, only depository institutions can access to reserve accounts and use it for interbank transactions through a real-time gross settlement(RTGS) system. In this respect, introducing central bank digital currency(hereafter CBDC) amounts to provide consumers an opportunity of holding a reserve account with the central bank directly. This "central bank digital money for all" idea is proposed and promoted explicitly in the recent literature. Barrdear and Kumhoff (2018) refer to a retail CBDC by meaning that "the central bank grants universal, electronic, 24x7, national-currency-denominated and interest-bearing access to its balance sheet." Bordo and Levin (2017) provides a way of implementing CBDC as "such accounts could be held directly at the central bank itself or made available via public-private partnerships with commercial bank". Berentsen and Schar (2018) discuss the advantages of introducing CBDC for all, which includes that the monetary policy becomes more transparent.[2] For example, since the central bank could simply use the interest on reserves as its main tool, the lowest interest rate will apply to the public as the same as to the depository institutions face and the interest payments are equally distributed to all.

Nevertheless, there are still many questions that need to be carefully addressed before this proposal is implemented. Specifically, this proposal needs to be examined in a perspective of the monetary policy framework such as channel or floor system, because the effect of monetary policy may vary by the amount of excess reserves if the reserves is allowed to be held by the public. In response to the last Great Recession, the Federal Reserve System has implemented various types of unconventional monetary policy with asset-purchase programs. As a result the excess reserves held in the banking sector had grown substantially, and the Federal Reserve began paying interest on reserve balances and shifted the monetary policy framework from the channel system to the floor system in October 2008. Afterwards, as a policy normalization plan the Federal Reserve announced to put a priority on a lift-off before unwinding in October 2019, so the level of reserves remains ample.[3] Therefore, introducing CBDC, especially in this floor system, provide a challenge

[1]In this paper we will concentrate on account-based CBDC in which retail consumers can access to central bank reserve accounts, and avoid the discussion on other forms of CBDC such as token-based CBDC and synthetic CBDC.

[2]all transactions are operated in a payment infrastructure which is associated with the central bank, so no monitoring is needed. reduce cost for monitoring commercial banks. Interest on reserves are paid only to the few depository institutions that have access to central bank reserves. CBDC can avoid this political issue in a simple way. CBDC would have a discipling effect on commercial banks since they need to attract bank deposits.

[3]In September 2014 the Federal Reserve considered two approaches: The first approach is a lift-off, which adjusts the interest on reserves to move the federal funds rate into the target. The second approach is unwinding, which reduces the Federal Reserve's balance sheet size by using an overnight reverse repurchase (ON-RRP) agreement facility. See Williamson (2015) for more detail.

to policy makers, because we have limited experience with the large excess reserves(hereafter LER) and have not considered seriously the central bank "open to all" yet in our monetary history.

In this paper I pay attention to the liquidity insurance of the banking sector, which is one of their primary functions. When the consumers are exposed to an idiosyncratic liquidity shock, it would be optimal to distribute liquidity by type. For instance, if one type of consumers require cash and/or CBDC for retail transactions while the other type of consumers can use any asset holdings including reserves and government bonds for collateral transactions, then it is efficient to provide currency/CBDC to the former type and the rest of assets to the latter one. If the consumers can access to reserve accounts at any time and use CBDC, then the reserves can be used in both types of transactions. Thus, if the whole banking sector's balance sheet is filled with the large excess reserves to the extreme, it would be nearly impossible to distinguish their types under anonymity. This difficulty of revealing types could reduce the bank's demand for reserves, while raise the demand for less liquid assets such as government bonds that cannot be used in retail transactions. The monetary policy is implemented by changing the supplies of liquid and illiquid assets such as reserves and government bonds in channel system or by adjusting the relative price between these assets in floor system. Thus, when the banks choose their asset portfolio, this change in the demand for assets can also make an impact on the monetary policy implementation. In this respect it will be meaningful to analyze the effect of introducing CBDC in each monetary policy regime by considering the banks' asset portfolio choice for liquidity insurance.

In order to study this issue I develop a search-theoretical monetary model a la Lagos and Wright (2005) and Rocheteau and Wright (2005) with a Diamond and Dybvig (1983) type banking contract shown in Sanches and Williamson (2010) and Williamson (2012, 2016). This model has an advantage of incorporating limited commitment and private information frictions in a simple way and is also highly tractable with insurance contracts and an array of assets. In the model agents can produce consumption goods with an elastic labor supply, but cannot consume by themselves. Under limited commitment and lack of record-keeping, agents need a medium of exchange to trade for consumption: one type of agents can use only currency and CBDC based on reserve accounts, while the other type can use the whole asset portfolio including reserves, government bonds and private assets.[4] The lack of memory assumption not only makes assets essential for transactions, but also keeps the type information of the agents secret.[5] An insurance contract with truth-telling incentive constraints arises to reveal the types by offering different types of assets. After announcing their type and receiving the assets, each agent has an opportunity to access their reserve accounts in the central bank.[6] The central bank issues currency and reserves by holding

[4]In the full-fledged model we introduce a Lucas tree with a fixed supply to consider an aggregate liquidity effect associated with the real interest rates.

[5]If record-keeping is available, credit and/or taxation can help to achieve the optimal equilibrium allocation even under private information.

[6]In equilibrium the agents do not change their asset portfolio, but this opportunity itself can affect their ex ante

government bonds and can provide a positive interest on reserves in the floor system.

There could be three equilibrium regimes according to the monetary policy variables, that is, channel system, floor system (without the LER), and floor system with the LER. In the channel system there are no excess reserves. The monetary policy adjusts the supplies of reserves and government bonds through open-market operations(hereafter OMOs) to control the liquidity premium of each asset separately and to affect the real interest rates. OMOs represents a conventional monetary policy tool to affect the real allocations by exchanging reserves with government bonds. For instance, an open-market purchase, injecting reserves and absorbing government bonds, raises the liquidity premium of government bonds, so the real interest rate would decrease. This mechanism will be maintained and work well when account-based CBDC is introduced. Since both cash and CBDC are generated from reserve accounts, although cash is replaced by CBDC and nothing will be changed.

In the floor system, OMOs are no longer effective because there is no return dominance between reserves and government bonds with excess reserves. Therefore, the real allocations are determined solely by the level of the interest on reserves, and adjusting the interest rate on reserves can make the same effect as OMOs in the channel system. In this case the public access to the reserve accounts does not make any changes as long as the types can be separated well. The agents who will participate in retail transactions would choose an offer with reserves only to withdraw cash or to use CBDC, while the agents who will engage in collateral trades would choose the asset portfolio including government bonds.

If the excess reserves are sufficiently large, one of the truth-telling incentive constraints could bind as agents can access to reserve accounts. In this new regime, the floor system with the LER, the OMOs can be effective. Since it is difficult to reveal the types with large excess reserves, government bonds are more useful than reserves. Thus, OMOs can relax or tighten the binding truth-telling constraint and affect the real interest rates. Additionally, the assets are used not only for medium of exchange, but also for revealing the types in this case. Thus, the liquidity premium of one asset can depend on the conditions of the corresponding asset market and the other asset market. For example, reserves are useful for the collateral transactions, but also can tighten the binding truth-telling constraint because it can be used as CBDC in retail transactions. Therefore, the liquidity premium of reserves reflects the frictions in both trading markets as a weighted average. More importantly, this floor system with the LER is suboptimal in the perspective of the aggregate liquidity supply. With the large excess reserves, the agents for retail transactions may pretend to be an agent who will use reserves as collateral, and use CBDC with their reserve accounts afterwards. Thus, the insurance contract has to provide less reserves to the agents for collateral trades inevitably and the efficiency of the asset transactions in this economy is lowered.

These results can provide some implications for introducing account-based CBDC in mone-

demands for the assets.

tary policy implementations. First, the effectiveness of OMOs relies not only on the liquidity of the assets that are exchanged, but also on the convertibility of the illiquid asset to the liquid asset or vice versa. Both reserves and government bonds are used for the same type of collateral trade, but they are not perfect substitutes. If reserve accounts are accessible, then reserves can be converted to CBDC and used for retail transactions. Therefore, the liquidity premia of reserves and government bonds could be different and OMOs can be effective. Second, if the truth-telling constraint binds with the large excess reserves, the effect of raising the level of interest on reserves on the real interest rates depends on the amount of excess reserves. Therefore, the amount of excess reserves must be considered even though the level of the interest on reserves is used as a main implementation tool in the floor system. Finally, the floor system with the LER is suboptimal in a respect of aggregate liquidity and welfare, although OMOs are effective. Therefore, reducing the amount of excess reserves by using ON-RRP before allowing the access to reserve accounts can be helpful to improve welfare.

1.1 Related Literature

There has been a growing literature on CBDC including its motivations and implications.[7] Specifically, Sanches and Keister (2019) and Williamson (2019a) construct a model in which CBDC can replace either cash or deposit, and evaluate the welfare effect along with its negative impact on financial intermediation and the cost for central bank independence, respectively. Andolfatto (2018) and Chiu et al. (2019) develop models in which an interest-bearing CBDC competes with bank deposits show that the introduction of CBDC need not disintermediate banks by reducing their lending, if the banks' market power in the deposit market is limited by CBDC.[8] On the other hand, in this paper CBDC is assumed to compete only with cash, because we focus on the efficiency of distributing liquidity when the people can use CBDC with their own reserve accounts.

There is also a number of literature on monetary policy with excess reserves in floor system. Since the interest on reserves can be used as a policy tool independently, Goodfriend (2002), Ennis and Weinberg (2007), and Keister et al. (2008) point out that the central bank can have an additional degree of freedom to choose the quantity of reserves by providing the interest on reserves.[9] Recently, Ireland (2014) and Ennis (2018) investigated the decoupling between the supply of reserves and the price level given the interest on reserves: Ireland (2014) finds out that the quantity of reserves can affect the nominal variables when there exists a cost for managing reserves. Similarly,

[7] For example, see Barrdear and Kumhoff (2018), Bech and Garratt (2017), Bordo and Levin (2017), Broadbent (2016), Dyson and Hodgson (2017), Engert and Fung (2017a), Engert and Fung (2017b), Raskin and Yermack (2016), Ricks et al. (2018), Brunnermeier and Niepelt (2019), and Kim and Kwon (2019)

[8] Andolfatto (2018) considers a type of monopoly bank and shows that CBDC would have no impact on bank lending if they can borrow from the central bank. Chiu et al. (2019) considers a Cournot competition in the deposit market and shows that both deposits and loans can be more created because an interest-bearing CBDC reduces their market power.

[9] In this respect Kashyap and Stein (2012) argue that the interest on reserves can be useful for aiming another target such as financial stability.

Ennis (2018) shows that the link between the supply of reserves and prices can be tightened again when the bank capital constraint binds with a large balance sheet. In my model the price variables are determined by the government budget constraint rather than the supply of reserves, which is similar to Cochrane (2014).[10] However, the supply of reserves can be used as a policy variable because the accessibility to reserve accounts provides an opportunity to use reserves directly as CBDC.

In terms of the (in)effectiveness of the monetary policy, the results of this paper are consistent with the literature. The ineffectiveness of monetary policy in this paper is close to the liquidity trap in Wallace (1981) rather than Williamson (2012) and Rocheteau et al. (2018). Since excess reserves and government bonds can be used for the same type of transactions, open-market-operations are irrelevant at any positive interest rate on reserves. In Williamson (2012) and Rocheteau et al. (2018), the liquidity trap arises only when the rates of return on money and government bonds are equal with the zero nominal interest rate.

In the model when the truth-telling incentive constraint binds with a sufficiently large excess reserves, monetary policy is less effective. This result is similar to Agenor and Aynaoui (2010) and Bech and Klee (2011), but the mechanism is different from their models. Agenor and Aynaoui (2010) show that given a precautionary demand for excess reserves, contractionary monetary policy can be less effective with prevailing excess reserves. Bech and Klee (2011) build a limited participation model with over-the-counter market to show that the federal funds rate rises less when the interest on reserves increases. In Armenter and Lester (2017) the interest rate spread in a corridor system can have a real effect because the spread between the two rates determines the trading gain and the efficiency of matching in their model.

This paper is also related to the literature that studies the bank's optimal decision with the interest on reserves and the excess reserves. Dressler and Kersting (2015) and Martin et al. (2016) focus on the bank's lending behavior given the excess reserves, while Dutkowsky and VanHoose (2013) and Dutkowsky and VanHoose (2017) study the bank's decision on sweeping and the size of bank balance sheet given the interest on reserves. In this paper, banks provide liquidity insurance given idiosyncratic liquidity risk as shown in Williamson (2016, 2019b), but I focus more on how introducing CBDC with the LER can affect the bank's decision on liquidity allocation under private information and the effectiveness of the monetary policy.[11]

For the welfare evaluation between channel system and floor system, Berentsen et al. (2014) show that there is a welfare improvement in channel system when lending is costly, because banks will hold more reserves *ex ante*, which can internalize the pecuniary externality in holding reserves. Williamson (2016) shows that the welfare can improve in the floor system by injecting

[10]Cochrane (2014) shows that when fiscal policy is restricted, the inflation rate can be determined by the government budget constraint although reserves and government bonds are perfect substitutes.

[11]Williamson (2019b) shows that the effect of raising the interest on reserves is different from the effect of reducing the central bank balance sheet by constructing a two-sector banking model with interbank markets.

more reserves given the interest on reserves, when reserves are more useful than government bonds in a liquidity aspect. In this paper, the channel system without excess reserves is preferred to the floor system in a perspective of aggregate liquidity supply, because the monetary policy is more effective when the trading markets are sufficiently segregated.

Finally, this paper is related to some papers that study the liquidity premium of illiquid assets. Herrenbrueck and Geromichalos (2017) show that the price of illiquid assets has a liquidity premium if the illiquid assets can be traded to obtain liquid assets. In this paper when the interest on reserves is implemented, the liquidity premium of the reserves decreases and the liquidity premium in the government bonds goes up, because the less liquid government bonds are more useful for revealing private information as the reserves are more easily convertible to currency. Therefore, when the truth-telling incentive constraint binds, injecting illiquid assets can be beneficial. In a respect of the social benefit of illiquid assets, the result of this paper can be interpreted along with Kocherlakota (2003) and Shi (2008). Kocherlakota (2003) shows that illiquid assets are beneficial because agents can trade liquid and illiquid assets after observing idiosyncratic shock. Shi (2008) shows that legal restriction on government bonds improves welfare when low marginal utility type cannot trade with bonds.

2 The environment

The basic model structure is based on Rocheteau and Wright (2005) in which *ex ante* heterogeneous agents trade in bilateral meetings and rebalance their portfolios in the centralized market. Time is discrete over infinite horizon and each period is divided into two sub-periods - the Centralized Market (CM) followed by the Decentralized Market (DM). There is a continuum of buyers, sellers and bankers, each with unit mass. An individual buyer has preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t [-H_t + u(x_t)]$$

where $H_t \in \mathbb{R}$ is the labor supply of the buyer in the CM, $x_t \in \mathbb{R}_+$ is the consumption of the buyer in the DM, and $0 < \beta < 1$. Assume that $u(\cdot)$ is strictly increasing, strictly concave, and twice continuously differentiable with $u^{'}(0) = \infty$, $u(0) = 0$, and $-x\frac{u^{''}(x)}{u^{'}(x)} = \gamma < 1$ for all $x > 0$.[12] For the proofs, a specific utility function $u(x) = \frac{x^{1-\gamma}}{1-\gamma}$ is used. Each seller has preferences as

$$E_0 \sum_{t=0}^{\infty} \beta^t [X_t - h_t]$$

[12]If the coefficient of relative risk aversion is greater than one, the asset demand will be strictly decreasing in the rate of return on the asset in this model.

where $X_t \in \mathbb{R}$ is the consumption of the seller in the CM, and $h_t \in \mathbb{R}_+$ is the labor supply of the seller in the DM. All the agents can consume and produce in the CM. But in the DM only buyers can consume and only sellers can produce. One unit of labor inputs can produce one unit of perishable consumption goods in either the CM or the DM.

In the CM all agents meet together and then production and consumption occur. Buyers receive a lump-sum transfer from the government, and the share holders of the Lucas trees receive the realized dividends. The previous debts are paid off, and assets and consumption goods are traded in a Walrasian market.

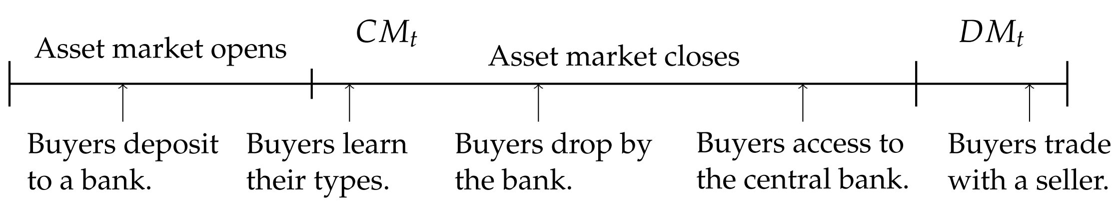
In the DM each buyer meets each seller randomly, and the terms of trade are determined by bargaining in the bilateral meeting. For simplicity, we assume that the buyer makes a take-it-or-leave-it offer to the seller in the meetings. Basically, there is no record-keeping technology, so the buyers and the sellers cannot verify the trading history of their partners in the DM. Moreover, no one can be forced to work under limited commitment. Thus, recognizable assets are essential for transactions in the DM and all of the trades must be *quid pro quo*. In a manner similar to Sanches and Williamson (2010) and Williamson (2012), there are two types of DM meetings. In a ρ proportion of DM meetings, which represents retail transactions, buyers will meet a seller who accepts only currency and CBDC. In the rest of DM meetings buyers will meet a seller who accepts the claims for the whole asset portfolio as collateral. In the following We refer these buyers as type 1 and type 2 buyers, respectively.

At the beginning of the CM, buyers do not know which type of match they will be in the DM. Given this idiosyncratic shock, buyers can write an insurance contract with banks. After the Walrasian market is closed in the CM, buyers learn their types and their types are private information. Once the type information is realized, each buyer can contact with his or her bank. Finally, buyers can access their reserve accounts in the central bank at the end of the CM.

There are an array of assets such as currency, CBDC, reserves and government bonds, which are the liabilities of both the fiscal authority and the central bank, in this economy. The fiscal authority can issue government bonds and provide nominal interests by collecting lump-sum taxes from buyers. One unit of government bonds trades at price q_t in terms of currency in the period t CM and pays one unit of currency in the period $t+1$ CM. The central bank can issue currency, CBDC, and reserves by purchasing government bonds. One unit of currency sells at price ϕ_t in terms of consumption goods in the period t CM and provides no interest. While currency is a physical object produced by the central bank, CBDC and reserves are account balances in the central bank. We define CBDC as an amount of reserve balances that is used in the ρ proportion of DM meetings. One unit of CBDC yields the CBDC holder in the next CM a total of R_t^d units of CBDC where R_t^d represents a gross nominal interest on CBDC. One unit of reserves, if the ownership is not changed in the DM, earns a total of $R_t^d R_t^m$ units of reserves in the next CM, where R_t^m represents an additional gross nominal interest on reserves. As CBDC is also an account balance, we assume that the central bank can pay a non-negative interest on CBDC less than or equal to the interest on reserves, $1 \leq R_t^d \leq R_t^d R_t^m$.[13] Moreover, we assume that CBDC transactions are not traceable as the same as currency, because we focus on the effect of using reserves accounts in retail market rather than replacing currency with CBDC itself.[14]

In the model currency and CBDC are substitutable because both assets can be used in the retail transactions and converted into the reserves immediately. Therefore, buyers would use CBDC if it provides a strictly positive nominal interest and use both cash and CBDC if the nominal interest rate is zero. However, reserves and government bonds are quite different, because buyers can use reserve balances as CBDC in the retail transactions, although both assets are useful in the $1-\rho$ proportion of transactions as collateral.[15]

Timing is described in Figure 1. At the beginning of the CM, government debt holders receive a unit of currency by redeeming a unit of government bonds. Buyers receive lump-sum transfers(or pay lump-sum taxes). Then all buyers and sellers provide labor and trade assets in a Walrasian market. Buyers deposit consumption goods and/or assets with a bank in exchange for an insurance contract. After the liquidity shock is realized, buyers learn their types and ρ proportion of buyers can withdraw currency and/or receive CBDC from the bank. Then, the buyers can access to the central bank before moving to the DM. In the DM buyers meet sellers randomly in the bilateral meeting and can trade with a take-it-or-leave-it offer.



[Figure 1. Time-line]

2.1 Banks

Given the idiosyncratic liquidity risk, a banking contract arises endogenously to allocate currency and CBDC and the other assets across the types of buyers. Without this insurance arrangement, type 1 buyers could run out of currency/CBDC and holding idle government bonds while type 2 buyers could hold low-yielding currency/CBDC instead of other high-yielding assets. Therefore, the banking contract can provide liquidity insurance by allocating currency/CBDC to type

[13]The interest rates on CBDC and reserves cannot be negative in this model, because if the central bank sets a negative interest on reserves, then all the buyers will hold currency instead of CBDC and reserves for the higher rate of return.

[14]Technically, it is possible to choose whether to monitor CBDC transactions or not as discussed in Bech and Garratt (2017). However, we assume that CBDC is a digital entry that can be transferred under anonymity.

[15]Another difference between reserves and government bonds is that the gross nominal interest on reserves, R_t^m, can be set by the central bank as a policy variable, while the price of government bonds, q_t, is determined in the market.

1 buyers and the rest of assets to type 2 buyers. Given the perfect competition, any agents can suggest an optimal banking contract that maximizes the expected utility of buyers and play a role as banks.

Under private information the banks cannot verify the type of individual buyers, so one type of buyers can mimic the other type of buyers. In general this private information friction does not matter because the reserves cannot be accessed by the buyers as shown in Sanches and Williamson (2010) and Williamson (2016). However, if CBDC is introduced and buyers can access to their reserve accounts additionally, then it is difficult for banks to reveal the types by providing different types of assets. For example, type 1 buyers are willing to mimic type 2 buyers for receiving a sufficient amount of reserves, because they can use their reserve balances as CBDC in the retail transactions. Therefore, given the excess reserves, the banks are required to adjust the amount of assets for both types of buyers to separate the types efficiently, and thus the effect of monetary policy could be changed.

We assume that all the claims issued by the banks and the central bank are not counterfeitable and buyers can meet only one bank after their liquidity shock is realized to prevent the banking contract from being unraveled.[16]

2.2 Government

At $t = 0$ government bonds are issued by the fiscal authority, and then currency, CBDC and reserves are injected by the central bank through open-market purchases. The initial revenue of issuing currency, CBDC, reserves, and government bonds is transferred to buyers. After $t = 0$, outstanding currency, CBDC, reserves and government bonds amounts can be supported by collecting taxes or providing transfers over time. So the consolidated government budget constraints can be written as

$$\phi_0(C_0 + D_0 + M_0 + q_0 B_0) = \tau_0 = V$$

and

$$\phi_t\{C_t - C_{t-1} + D_t - R^d_{t-1} D_{t-1} + M_t - R^m_{t-1} M_{t-1} + q_t B_t - B_{t-1}\} = \tau_t,\ t = 1, 2, 3, \ldots$$

where C_t, D_t, M_t and B_t denote the nominal quantities of currency, CBDC, reserves and government bonds held by the private sector in the CM at time t, respectively. In the model the central bank can buy and sell the private assets such as the Lucas trees.[17] τ_t denotes the real value of the

[16] If *ex post* trading among the buyers is allowed then the equilibrium with the insurance contract can be unraveled and collapsed into an asset-trading market equilibrium as shown in Jacklin (1987). Note that this equilibrium with the insurance contract provides higher welfare than the asset-trading market equilibrium in general: Given the utility function, $u(x) = \frac{x^{1-\gamma}}{1-\gamma}$, both equilibrium allocations are equally efficient only when $u(x) = ln(x)$ with $\gamma = 1$.

[17] Note that there is no equilibrium case where only currency is scarce while the other assets are plentiful as shown in Champ et al. (1996), since the central bank is allowed to purchase private assets in this model.

lump-sum transfer from the fiscal authority to each buyer in the CM at period t. As described in Williamson (2012,2016), we assume that the fiscal authority keeps the total value of the outstanding consolidated government debt as a constant, V, after the fixed amount, τ_0, is transferred at $t = 0$. This fiscal rule plays a role as separating fiscal policy and monetary policy. The fiscal policy manages the total quantity of government debt, and the monetary policy adjust the composition of the debt by exchanging liquid assets such as currency, CBDC, reserves with government debt. Since this fiscal rule restricts the total supply of public safe assets as scarce, in the model we analyze the monetary policy effect when the fiscal policy is limited.[18]

On the other hand, the amount of private liquidity in this economy can vary by the price level of the Lucas tree. Since the real rate of return on the Lucas tree is negatively related to its equilibrium price level in this model, the aggregate liquidity supply could increase as the real interest rate falls. With this setting, we can observe the effect of monetary policy on aggregate liquidity in the private sector.

In every period to maintain the real value of outstanding consolidated government debt, the real term of lump-sum transfer τ_t is derived passively from

$$\tau_t = \underbrace{(1 - \frac{\phi_t}{\phi_{t-1}})V}_{seigniorage} + \underbrace{\frac{\phi_t}{\phi_{t-1}}\{(1 - R^d_{t-1})\phi_{t-1}D_{t-1} + (1 - R^m_{t-1})\phi_{t-1}M_{t-1} + (q_{t-1} - 1)\phi_{t-1}B_{t-1}\}}_{real\ interest\ payment}, \ t = 1, 2, 3, \ldots.$$

Note that the lump-sum transfer consists of seigniorage from inflation, real interest payments for government bonds and reserves, and investment earning from the Lucas tree.

3 Maximization Problem

Under perfect competition a representative bank suggests an insurance contract to maximize the buyer's ex ante expected utility. In this respect, the buyer's problem is trivial because it is solved by the bank. Moreover, the seller's problem is also trivial since sellers always accept the buyer's take-it-or-leave-it offer in equilibrium. Thus, to construct an equilibrium, we focus on the bank's maximization problem and asset market clearing conditions. A representative bank solves the following problem in the CM of period t:

$$\underset{k_t, c_t, d_t, m_t, b_t, x_{1t}, x^m_{2t}, x^b_{2t}}{Max} - k_t + \rho u(x_{1t}) + (1 - \rho)u(x^m_{2t} + x^b_{2t}) \tag{1}$$

[18] As discussed in Ennis (2015), this assumption requires the fiscal authority to keep reacting to the monetary policy in order to maintain the real value of the government debt constant, and it is a crucial point to affect the real allocations. However, without the monetary policy, the fiscal policy itself is ineffective in the real allocations as long as the real value of government debt is maintained. In this respect, we can interpret the changes in real allocations as the monetary policy effect.

subject to the participation constraint of the bank,

$$
\begin{aligned}
&k_t - c_t - d_t - m_t - q_t b_t + \{\tfrac{\beta\phi_{t+1}}{\phi_t} c_t + \tfrac{\beta\phi_{t+1}}{\phi_t} R_t^d d_t - \rho x_{1t}\} \\
&+\{\tfrac{\beta\phi_{t+1}}{\phi_t} R_t^d R_t^m m_t - (1-\rho) x_{2t}^m\} + \{\tfrac{\beta\phi_{t+1}}{\phi_t} b_t - (1-\rho) x_{2t}^b\} \geq 0
\end{aligned} \tag{2}
$$

and the currency/CBDC, reserves and assets constraints,

$$\frac{\beta\phi_{t+1}}{\phi_t} c_t + \frac{\beta\phi_{t+1}}{\phi_t} R_t^d d_t - \rho x_{1t} \geq 0, \tag{3}$$

$$\frac{\beta\phi_{t+1}}{\phi_t} R_t^d R_t^m m_t - (1-\rho) x_{2t}^m \geq 0, \tag{4}$$

$$\frac{\beta\phi_{t+1}}{\phi_t} b_t - (1-\rho) x_{2t}^b \geq 0, \tag{5}$$

and the truth-telling constraints,

$$R_t^m x_{1t} \geq x_{2t}^m, \tag{6}$$

$$x_{2t}^m + x_{2t}^b \geq R_t^m x_{1t}, \tag{7}$$

and non-negative constraints,

$$k_t, c_t, d_t, m_t, b_t, x_{1t}, x_{2t}^m, x_{2t}^b \geq 0. \tag{8}$$

The problem (1) subject to constraints (2)-(8) states that a banking contract is chosen in equilibrium to maximize the expected utility of the representative buyer subject to the participation constraint for the bank (2) and the resource constraints for currency/CBDC, reserves and private assets (3)-(5) and the truth-telling incentive constraint for type 1 and 2 buyers, respectively, (6)-(7) and non-negativity constraints (8). In (1)-(8) k_t denotes deposit of buyers, c_t, d_t, m_t, and b_t denote the quantities of currency, CBDC, reserves and government bonds in terms of the CM good in the period t held by banks. x_{1t} denotes the consumption of type 1 buyers in the period t DM, while x_{2t}^m and x_{2t}^b denote the consumption of type 2 buyers in the period t DM based on reserves and the rest of assets, respectively, and we define x_{2t} as the sum of x_{2t}^m and x_{2t}^b.

The participation constraint (2) implies that the net payoff for the bank must be non-negative. In the period t CM, the bank receives k_t deposits and invest in assets, and then provides x_{1t} amount of currency/CBDC to type 1 buyers at the end of the CM and x_{2t} amount of consumption goods to agents who hold the deposit claims in the next period $t+1$ CM. The currency/CBDC, reserves, and assets constraints (3)-(5) represent the resources of the bank. Truth-telling incentive constraints (6)-(7) imply that each type of buyers prefers their own offer to the offer for the other type. Since buyers can access to their reserve accounts, type 1 buyers can receive reserves and other assets by mimicking type 2 buyers, and use CBDC in the retail market. Similarly, type 2 buyers can receive currency/CBDC by mimicking type 1 buyers and deposit it to the central bank as reserves.

From now on I focus on stationary equilibrium without time scripts on variables where $\frac{\phi_{t+1}}{\phi_t} = \frac{1}{\mu}$ holds for all time t and μ denotes the gross inflation rate. From the maximization problem, the first-order conditions can be derived as

$$\frac{\mu}{\beta R^d} = u'(x_1) + \frac{R^m(\lambda_1 - \lambda_2)}{\rho}, \tag{9}$$

$$\frac{\mu}{\beta R^d} = u'(x_2^m + x_2^b)R^m - \frac{R^m(\lambda_1 - \lambda_2)}{1-\rho}, \tag{10}$$

$$q\frac{\mu}{\beta} = u'(x_2^m + x_2^b) + \frac{\lambda_2}{1-\rho}, \tag{11}$$

where λ_1 and λ_2 denote each multiplier associated with the constraints (6)-(7), respectively. In equilibrium asset markets clear in the CM with

$$\begin{aligned} c &= \phi C, \\ d &= \phi D, \\ m &= \phi M, \\ b &= \phi B. \end{aligned} \tag{12}$$

Since the supplies of currency, CBDC and reserves are equal to the central bank's government bond holdings, we have

$$c + d + m = (V - qb), \tag{13}$$

and the proportion of currency, CBDC and reserves among the supply of the total assets can be defined as δ by

$$c + d + m = \delta V. \tag{14}$$

I assume that the central bank can set the interests on reserves and CBDC, $R^d R^m$ and R^m, and choose the proportion of currency, CBDC and reserves in the total asset supply, δ, to implement monetary policy.

Finally, the quantity of government bonds held by the private sector must be less than or equal to the total government bonds issued by the fiscal authority as

$$0 \le qb \le V. \tag{15}$$

Definition 1. *Given parameters (ρ, y, V) and the policy variables (R^d, R^m, δ), a stationary monetary equilibrium consists of quantities (x_1, x_2^m, x_2^b) and prices (μ, q) and multipliers λ_1 and λ_2 which solve equations (9)-(15).*

Since a quasi-linear utility is adopted in the model, the rates of return on assets such as currency, $\frac{1}{\mu}$, CBDC, $\frac{R^d}{\mu}$, reserves, $\frac{R^d R^m}{\mu}$, government bonds, $\frac{1}{q\mu}$, and the Lucas tree, $1+\frac{y}{\psi}$, cannot exceed the inverse of the time preference, $\frac{1}{\beta}$. In the model if a truth-telling incentive constraint binds, the rate of return on reserves can be higher or lower than the rates of return on government bonds and the Lucas tree in equilibrium, although reserves are used for transactions as collateral the same as the government bonds and the Lucas tree. Thus, we have no-arbitrage conditions in equilibrium as

$$\begin{aligned} \frac{1}{\mu} \leq \frac{R^d}{\mu} \leq \frac{R^d R^m}{\mu} &\leq \frac{1}{\beta}, \\ \frac{1}{q\mu} = \frac{\psi+y}{\psi} &\leq \frac{1}{\beta}. \end{aligned} \tag{16}$$

In the following analysis we assume that the total supply of assets in this economy is sufficiently small as $V < x^*$, where x^* satisfies with $u^{'}(x^*) = 1$, in order to make the first-best allocation, i.e. $x_1 = x_2 = x^*$, infeasible.[19]

4 Monetary Equilibrium

In this section, we characterize the equilibrium allocations. Equilibrium cases can be distinguished by whether excess reserves exist or not, and also by which of the incentive constraints (6)-(7) bind or not.[20] In the following we describe channel systems and floor systems as different equilibrium cases and will compare the effect of monetary policy and the welfare in these equilibrium allocations.

4.1 Channel Systems

In a channel system the central bank can set the interests on CBDC and reserves, R^d and $R^d R^m$, lower than the nominal interest rate on government bonds, $\frac{1}{q} - 1$, which is controlled by open-market-operations, δ. So banks will not hold any excess reserves in equilibrium, $m = 0$ and $x_2^m = 0$, in this case. We also know that the constraints (3)-(5) always bind when the first-best allocation is not feasible. The first step of characterization is to solve for the equilibrium conditions without the binding incentive constraints (6)-(7). If $\lambda_1 = \lambda_2 = 0$, then the first-order conditions (9) and (11) can be reduced into

$$\frac{\mu}{\beta R^d} = u^{'}(x_1), \tag{17}$$

$$q\frac{\mu}{\beta} = u^{'}(x_2), \tag{18}$$

[19] If $V \geq x^*$ then we can always achieve $x_1 = x_2 = x^*$ by setting $z = 1$ and $\delta \geq \frac{\rho x^*}{V}$ in equilibrium, so there is no reason to consider monetary policy.

[20] In this paper, channel and floor systems are the different types of *equilibrium*, which can be distinguished by the amount of excess reserves in the banking sector.

respectively. By using these first-order conditions (18)-(19) and the binding constraints (3) and (5), the government budget constraint (13) can be transformed into a feasibility condition,

$$\rho x_1 u'(x_1) + (1-\rho) x_2 u'(x_2) = V. \tag{19}$$

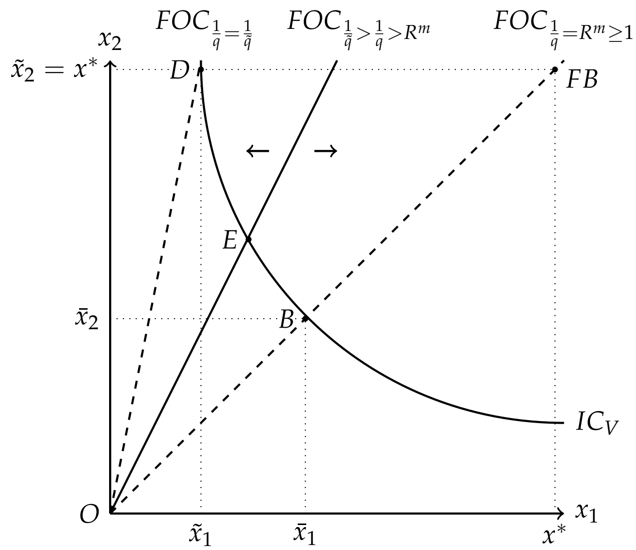
Note that given a nominal interest rate target $\frac{1}{q} - 1$, x_1 and x_2 have a strictly positive relationship as $qR^d u'(x_1) = u'(x_2)$ in (17)-(18) and a strictly negative relationship in (19). So there is a unique equilibrium allocation (x_1, x_2), which satisfies with (17)-(19): The point E in Figure 2 describes the equilibrium allocation (x_1, x_2) that intersects FOC curve from (17)-(18) and IC curve (19). In order to support this equilibrium allocation (x_1, x_2, q), the central bank can implement open-market-operations by choosing δ as

$$\delta = \frac{\rho x_1 u'(x_1)}{V}, \tag{20}$$

which can be derived from (14) and (19). Notice that the equilibrium allocation (x_1, x_2) is determined by the relative rate between the interest on CBDC, R^d, and the nominal interest rate, $\frac{1}{q}$. That means, when R^d varies, only the nominal interest rate and the inflation rate, μ, will change along with it and the other real allocations are maintained. Thus, without the loss of generality, we assume that the interest on CBDC is normalized as one, $R^d = 1$, from now on.

In Figure 2, given $R^m \geq 1$ the equilibrium allocations (x_1, x_2) are feasible on the IC curve between the points B and D associated with $\frac{1}{q} \in [R^m, \frac{1}{\tilde{q}}]$ where $\tilde{q}$ is defined as the lower bound of q at the point D. Since δ is increasing in x_1 in (20), each $q \in [\tilde{q}, \frac{1}{R^m}]$ is implemented by a corresponding $\delta \in [\tilde{\delta}, \bar{\delta}]$, where $\tilde{\delta}$ and $\bar{\delta}$ are the lower and upper bound of δ: Given the utility function, $u(x) = \frac{x^{1-\gamma}}{1-\gamma}$, $q = \left(\frac{1-\rho}{\rho}\frac{\delta}{1-\delta}\right)^{\frac{\gamma}{1-\gamma}}$ can derived from (17)-(20). As shown in Figure 2, when $(\tilde{x}_1, \tilde{x}_2)$ and $(\bar{x}_1, \bar{x}_2)$ are defined as the allocations at $q = \tilde{q}$ and $q = \frac{1}{R^m}$, respectively, each $\tilde{\delta}$ and $\bar{\delta}$ satisfies (20) with $\tilde{x}_1$ and $\bar{x}_1$, respectively.[21]

[21] Note that in case of the lower bound, $\frac{1}{q} = R^m$, $\delta > \bar{\delta}$ can also support the same equilibrium allocation with $\delta = \bar{\delta}$, because the excess reserves can be held at $\frac{1}{q} = R^m$.



[Figure 2. Equilibrium in Channel System]

Lemma 1. *In channel systems the incentive constraints (6)-(7) do not bind.*

Proof. *See the appendix.*

Lemma 1 shows that both the incentive constraints do not bind in the channel system. The insurance contract provides currency for type 1 buyers and government bonds for type 2 buyers to make the marginal rate of substitution between currency and government bonds equivalent to the given nominal interest rate. Without the excess reserves, type 1 buyers prefer currency/CBDC because it is unavailable to use government bonds in retail transactions. Type 2 buyers prefer government bonds as long as the rate of return in government bonds is greater than the rates of return on currency and CBDC. Therefore, the optimal insurance contract does not violate the truth-telling incentive constraints without excess reserves.

In a channel system, monetary policy is implemented only by open-market operations, because the level of the interest rate on reserves, R^m, is irrelevant to determine the equilibrium allocations. For example, suppose that the central bank tries to raise the nominal interest rate, $\frac{1}{q} - 1$, by reducing δ, i.e. open-market sales. Then, the currency is absorbed and the government bonds are injected into the market, so the liquidity premium on currency goes up whereas the liquidity premium on the government bonds falls. Thus, the consumption in the currency trade, x_1, falls while the consumption in the collateral transaction, x_2, rises.[22] Since the real rate of return on currency decreases, the inflation rate rises. Meanwhile, the real rate of return on government bonds increases. Therefore, the nominal interest rate also rises by the Fisher equation.[23]

[22] In Figure 2, the *IC* curve remains, while the *FOC* curve shifts to the left.
[23] This mechanism is the same as described in Williamson (2012,2016).

In channel system, open-market operations can be ineffective only at the lower bound, $\frac{1}{q} = R^m$. [24] Since the rates of return on reserves and government bonds are equal at $\frac{1}{q} = R^m$, we can have excess reserves, $m \geq 0$ and $x_2^m \geq 0$, in this case. The allocation (x_1, x_2) is determined from (17)-(18) with $\frac{1}{q} = R^m$, but reserves and government bonds are indeterminate in equilibrium from (13)-(14) and (19)-(20), $m + b = V - \rho x_1 u'(x_1)$. Thus, if $\delta > \bar{\delta}$, the OMOs is no longer effective.

In sum, if the monetary policy is implemented in channel system, there is no effect on real allocations when an account-based CBDC is introduced to the agents.

4.2 Floor Systems

In a floor system the interest on reserves must be greater than or equal to the nominal interest rate, $R^m \geq \frac{1}{q}$, to have excess reserves. Given the interest on reserves, R^m, if $\delta = \bar{\delta}$ then we have the same equilibrium as in channel systems with no excess reserves, $m = 0$ and $x_2^m = 0$, so when $\delta > \bar{\delta}$, banks hold strictly positive excess reserves on their balance sheets as $m > 0$ and $x_2^m > 0$. In order to implement floor systems, the central bank can set $R^m \geq 1$ and $\delta > \bar{\delta}$ where $\bar{\delta}$ satisfies with $R^m = \left(\frac{1-\rho}{\rho} \frac{\bar{\delta}}{1-\bar{\delta}} \right)^{-\frac{\gamma}{1-\gamma}}$, given the utility function $u(x) = \frac{x^{1-\gamma}}{1-\gamma}$.

Define α as a proportion of type 2 buyer's consumption supported by excess reserves, $\alpha = \frac{x_2^m}{x_2^m + x_2^b}$. Note that $\alpha \in [0,1]$ increases in δ and corresponds to $\delta \in [\bar{\delta}, 1]$ since $\alpha = \frac{\delta - \bar{\delta}}{1-\bar{\delta}}$ holds from the binding constraints (3)-(5) and (13)-(14).[25] Then the truth-telling constraint (6) can be rewritten as

$$R^m x_1 \geq \alpha x_2. \tag{21}$$

Lemma 2. *In the floor system, the truth-telling constraint (7) never binds, but the truth-telling constraint (6) can bind when the excess reserves are sufficiently large as $\delta > \hat{\delta}$ where $\hat{\delta}$ satisfies $\frac{\hat{\delta} - \bar{\delta}}{1-\bar{\delta}} = (R^m)^{1-\frac{1}{\gamma}}$.*

Proof. *See the appendix.*

Lemma 2 shows that the incentive constraint for type 1 buyers can bind when the excess reserves are sufficiently large. In a floor system, the interest on reserves plays the same role as the nominal interest rate in the channel system to adjust the marginal rate of substitution between currency/CBDC and government bonds. However, when CBDC is introduced, type 1 buyers can use CBDC by accessing to the reserve accounts. So, the optimal allocation cannot be obtained when the large excess reserves are provided to type 2 buyers, because type 1 buyers can mimic type 2 buyers.

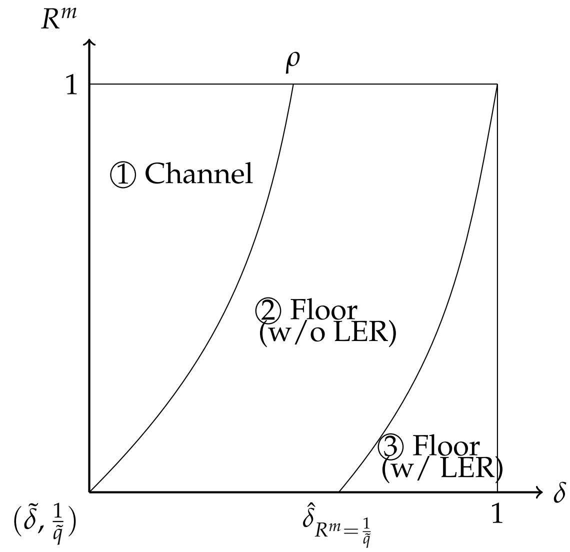
Notice that only type 1 buyers deviate in this model unlike Diamond and Dybvig (1983) where type 2 buyers deviate in a similar setting, because we assume that the coefficient of relative risk

[24] In this respect the equilibrium feature at the lower bound is similar to that in the floor system.

[25] Note that $c = \rho x_1 u'(x_1) = \bar{\delta} V$, $zm = \alpha(1-\rho) x_2 u'(x_2) = (\delta - \bar{\delta})V$, and $qb = (1-\alpha)(1-\rho)x_1 u'(x_1) = (1-\bar{\delta})V$ hold from (3)-(5) and (13)-(14).

aversion, γ, is less than 1. Since the insurance contract makes type 1 buyers worse off and type 2 buyers better off with $\gamma < 1$, type 1 buyers tend to deviate and the incentive constraint (6) is binding here. [26]

Therefore, we can describe our equilibrium cases as shown in Figure 3. Given $R^m > 1$, if $\tilde{\delta} \leq \delta \leq \bar{\delta}$ then we can conduct open-market operations in channel systems. Given $R^m > 1$, if $\delta > \bar{\delta}$ we need to implement monetary policy in floor systems. In case of $\bar{\delta} < \delta \leq \hat{\delta}$ then the incentive constraint (21) does not bind, but it binds in case of $\hat{\delta} < \delta \leq 1$. [27]



[Figure 3. Equilibrium cases]

Floor system without large excess reserves is exactly the same as the liquidity trap equilibrium in Williamson (2012).

4.3 Floor System with Large Excess Reserves

When $\lambda_1 > 0$, the feasibility condition (19) does not change, but the truth-telling constraint (21) binds with equality instead of the first-order conditions (17)-(18).[28] Thus, in this case given (R^m, δ), the equilibrium allocation (x_1, x_2) in the floor system with $\lambda_1 > 0$ is determined by (19) and (21) with equality. Define (x_1^i, x_2^i) as the equilibrium allocations in the channel system(c) and floor system(f), respectively, where $i = \{c, f\}$. As shown in Figure 4, given the same level of interest on

[26] If $\gamma > 1$, then type 2 buyers would deviate similar to Diamond and Dybvig (1983), and the main result still holds that the equilibrium allocation with the large excess reserves is inefficient.

[27] Note that given $(R^m, \bar{\delta})$, $\hat{\delta}$ can be derived from $(R^m)^{1-\frac{1}{\gamma}} = \frac{\hat{\delta}-\bar{\delta}}{1-\bar{\delta}}$. $\hat{\delta}_{\frac{1}{\bar{q}}=R^m} = (1-\tilde{\delta})\tilde{q}^{\frac{1}{\gamma}-1} + \tilde{\delta}$ could be either larger or smaller than ρ.

[28] Note that (19) does not change because λ_1 terms are cancelled out when the binding constraints (3)-(5) and the first-order conditions (9)-(11) are plugged into the government budget constraint (13).

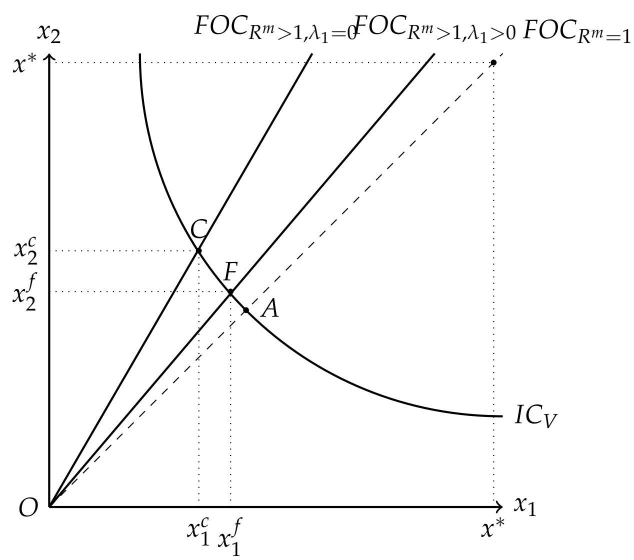
reserves, R^m, the equilibrium allocation (x_1^f, x_2^f) in the floor system with $\lambda_1 > 0$ can be described as the point F while the equilibrium allocation (x_1^c, x_2^c) in the channel system(or in the floor system with $\lambda_1 = 0$) can be shown as the point C. Notice that $x_1^c < x_1^f$ and $x_2^c > x_2^f$ in the graph, because (x_1^c, x_2^c) and (x_1^f, x_2^f) still satisfy with the same feasibility condition (19) while (21) violates at (x_1^c, x_2^c) because of $\lambda_1 > 0$. Given the large excess reserves, the bank cannot provide plentiful reserves to type 2 buyers because type 1 buyers could mimic them. Therefore, the bank would provide more currency/CBDC to type 1 buyers and less reserves to type 2 buyers.

Therefore, given the interest on reserves, R^m, the inflation rate and the real interest rate on government bonds can be changed from the ones in the channel system. When $\lambda_1 > 0$, from (9)-(11) we obtain

$$\frac{1}{\beta r_m^f} := \frac{\mu^f}{\beta R^m} = \frac{\rho u'(x_1^f)}{R^m} + (1-\rho)u'(x_2^f), \tag{22}$$

$$\frac{1}{\beta r_b^f} := q\frac{\mu^f}{\beta} = u'(x_2^f). \tag{23}$$

where μ^i, r_m^i, r_b^i denote the inflation rate, the rate of return on reserves, and the rate of return on government bonds in the channel system(c) and floor system(f), respectively, where $i = \{c, f\}$.[29]



[Figure 4. Equilibrium in Floor System]

In this case, the liquidity premia of reserves do not depend only on the collateral transactions market condition, because excess reserves can tighten the truth-telling constraint for the retail

[29]Note that μ^c, r_m^c, and r_b^c can be obtained in (17)-(18) as $\frac{1}{\beta r_m^c} = \frac{\mu}{\beta R^m}$ and $\frac{1}{\beta r_b^c} = q\frac{\mu}{\beta}$. Moreover, (22)-(23) can be collapsed into (17)-(18) in the case of $\lambda_1 = 0$, because $R^m u'(x_2) = u'(x_1)$ holds when $\lambda_1 = 0$ in equilibrium.

market transactions. Thus, the liquidity premium in reserves is calculated as a weighted average of the marginal utility in both types of trades as shown in (22).

Given $\lambda_1 > 0$, since the point C is not achieved, the marginal rate of substitution between type 1 buyer's consumption and type 2 buyer's consumption, $\frac{u'(x_1^f)}{u'(x_2^f)}$, is lower than the interest on reserves, R^m. It is optimal for the banks to raise x_2 and reduce x_1, but it is not feasible because they cannot reveal the types under private information. Therefore, there arises a positive liquidity premium on currency in (9) and a negative liquidity premium on reserves in (10) with $\lambda_1 > 0$. Moreover, the liquidity premium on government bonds is greater than the liquidity premium on reserves with $\lambda_1 > 0$ in (11), because government bonds are more useful than reserves to reveal the types. Therefore, in equilibrium, we have a return dominance between reserves and government bonds, $r_m^f > r_b^f$, as shown in (22)-(23) because $\frac{1}{q} < R^m$ and $R^m u'(x_2) > u'(x_1)$ holds when (21) binds. Thus, open-market operations, the exchange between reserves and government bonds, can be effective in this case. Notice that the nominal interest rate, $\frac{1}{q}$, is lower than the interest on reserves, R^m, in the model which actually happened in the U.S. federal funds market recently. In reality, the reason is based on a specific feature of the U.S. financial system, but our model shows that a similar phenomenon can occur when the reserve accounts are allowed to the public with large excess reserves.[30] When the people can use the reserves as CBDC, the reserves become more liquid. Thus, it can be difficult to distribute liquidity by types efficiently because the supply of illiquid assets such as government bonds is scarce with the large excess reserves. In this respect our result also emphasizes the role of illiquid assets to distribute liquidity efficiently as shown in Kocherlakota (2003).

4.3.1 Open-market Operations($\triangle\delta$)

Notice that we have two policy variables, the proportion of currency/CBDC and reserves, δ (or α), and the interest on reserves, R^m. Given R^m, if the proportion of currency and reserves, δ, is raised by open-market operations, the equilibrium allocation (x_1^f, x_2^f) moves toward the point C in Figure 4.

Proposition 1. *Given R^m, the inflation rate decreases in δ while the real interest rate on government bonds increases in δ in the floor system with LER.*

Proof. *See the appendix.*

In this case, there can be two different effects on the liquidity premia in currency and government bonds. Open-market purchases increase the quantity of reserves and decrease the quantity

[30] Afonso et al. (2013a) and Afonso et al. (2013b) explain the reason: In the U.S. the Federal Home Loan Bank system is not eligible to earn the interest on reserves, so they lend to the depository institutions that have reserve accounts in the federal funds market, and these institutions are subject to the balance sheet cost. As the balance sheet cost is significant, the gap between the federal funds rate and one-month Treasury bill rate is maintained for a long time.

of government bonds in the market. Since the truth-telling incentive constraint (21) is tightened further, the bank provides more currency/CBDC to type 1 buyers and less reserves to type 2 buyers.

Consequently, the liquidity premia on currency/CBDC and government bonds are reduced as the gap between type 1 and 2 buyers' consumption becomes smaller. Thus, given that the level of the interest on reserves is maintained, the inflation rate rises. The real interest rate falls as type 2 buyers' consumption decreases.

The opposite movement of the inflation rate and the real interest rate is similar to the Mundell-Tobin effect, in which capital investment is considered. By raising money supply, the inflation rate goes up and the real interest rate decreases, because capital investment is more demanded than currency trade. However, the reason that the retail transactions are restricted in this model is based on the scarcity of illiquid assets rather than the trade-off between the currency demand and capital investment.

4.3.2 Interest on Reserves($\triangle R^m$)

Given δ, if the interest rate on reserves, R^m, is raised, then the equilibrium allocation (x_1^f, x_2^f) also moves from point F toward point C in Figure 4 as the FOC curve (21) shifts to the left.

Proposition 2. *Given δ, both the inflation rate and the real interest rate on government bonds increase in the interest of reserves, R^m, in the floor system with LER.*

Proof. *See the appendix.*

When the interest on reserves goes up, the truth-telling constraint (21) is relaxed with higher λ_1. Therefore, the liquidity premium in currency goes up while the liquidity premium in government bonds decreases additionally in the floor system with LER. So, the real interest rate on government bonds increases further than it would in the floor system without LER.

The effects of monetary policy implementations on the inflation rate and the real interest rates by different cases are summarized in Table 1. In the channel system only the open-market operations are effective, while only adjusting the interest on reserves is effective in the floor system without LER. In the floor system with LER, both adjusting the interest on reserves and the open-market operations are effective. Moreover, the direction of the real interest rate is opposite in the case of open-market operations.

4.3.3 Effectiveness of the Interest on Reserves($\triangle R^m$)

Note that raising the interest on reserves in the floor system either without LER or with LER has a positive effect on the inflation rate and the real interest rates. However, the effectiveness of adjusting the interest on reserves must be different between these two regimes whether the

Table 1: Comparative Statics

Regime (Policy)	Channel (R^m, δ)	Floor(w/o LER) (R^m, δ)	Floor(w/ LER) (R^m, δ)
μ	(x,$-$)	(+,x)	(+,$-$)
r_m	(x,$-$)	(+,x)	(+,+)
r_b	(x,$-$)	(+,x)	(+,+)
x_1	(x,+)	($-$,x)	($-$,+)
x_2	(x,$-$)	(+,x)	(+,$-$)

incentive constraint binds or not. We can compare the effects of changing R^m on the inflation rate and the real interest rate, given the equilibrium allocations at the same level of R^m.

Proposition 3. *At the same level of δ, when the interest on reserves is raised, the inflation rate increases less and the real interest rate on the government bonds decreases more in the floor system with LER compared to the floor system without LER.*

Proof. *See the appendix.*

Proposition 3 shows that raising the interest on reserves in the floor system with LER can increase the real interest rate more than that in the floor system without LER. If the truth-telling incentive constraint binds, the constraint is relaxed with higher λ_1 when the interest on reserves goes up. Thus, it raises the inflation further in (9) and reduces the real interest rate on government bonds further in (11). That means, if CBDC is introduced, raising the interest on reserves can be more contractionary with the LER.

5 Equilibrium with Private Assets

In the real world, banks invest not only in nominal government debt such as currency, reserves, and government bonds, but also in real assets and/or loans. In this section I extend the model by introducing a divisible Lucas tree to study the effect of monetary policy on the aggregate private liquidity, which is associated with the asset prices.[31]

Suppose that a divisible Lucas tree with a fixed supply of A is endowed to buyers in the initial period, $t = 0$, CM and pays off y units of consumption goods as a dividend in the CM in every period and trades at the price of ψ_t in terms of goods in the period t CM.

Given the fixed supply of the Lucas tree, if the asset price goes up then aggregate liquidity can increase in this economy. to focus on the effectiveness of monetary policy without considering the effect on the aggregate liquidity supply through the price of the private assets. In the model the real quantity of public assets is assumed to be constant as V to restrict the fiscal policy effect, so

[31] In the model the real asset represents commodities, real estate and equipment in reality, which are less preferred in retail transactions, but used as collateral.

to know the effect on Agg. liquidity, introduce private asset. real interest rates decreases then the asset price goes up and liquidity increases. It can also be interpreted as loans, because the real

The maximization problem can be modified as

$$\begin{aligned} &k_t - c_t - d_t - m_t - q_t b_t - \psi_t(1-\theta_t)a_t + \{\tfrac{\beta\phi_{t+1}}{\phi_t}c_t + \tfrac{\beta\phi_{t+1}}{\phi_t}R^d_t d_t - \rho x_{1t}\} \\ &+\{\tfrac{\beta\phi_{t+1}}{\phi_t}R^d_t R^m_t m_t - (1-\rho)x^m_{2t}\} + \{\tfrac{\beta\phi_{t+1}}{\phi_t}b_t + \beta(\psi_{t+1}+y)(1-\theta_t)a_t - (1-\rho)x^b_{2t}\} \geq 0 \end{aligned} \tag{24}$$

$$\beta(\psi_{t+1}+y)(1-\theta_t)a_t - (1-\rho)x^b_{2t} \geq 0, \tag{25}$$

$$\frac{\psi}{\beta(\psi+y)} = u^{'}(x^m_2 + x^b_2) + \frac{\lambda_2}{1-\rho}, \tag{26}$$

$$a = A, \tag{27}$$

$$c + d + m = (V - qb) + \psi\theta a, \tag{28}$$

where θ_t denotes a proportion of the Lucas trees purchased by the central bank and a_t denotes the demand for the asset holdings of the bank and ψ_t. [32]

Given $A > 0$, instead of (19), we can obtain

$$\rho x_1 u^{'}(x_1) + (1-\rho)x_2 u^{'}(x_2) = V + \psi A. \tag{29}$$

where $\psi = \frac{\beta y u^{'}(x_2)}{1-\beta u^{'}(x_2)}$ from the binding constraints (3) and (5), government budget constraint (13) and the first-order conditions (17)-(18) in the channel system. In the floor system, even with $\lambda_1 > 0$, we still have (24) from (3)-(5), (13) and the first-order condition (21). Therefore, the equilibrium allocation (x_1, x_2) is uniquely determined from (17)-(18) and (24) in the channel system and the floor system without LER, while determined from (21) and (24) in the floor system with LER.

Notice that the proportion of private assets held by the central bank, $\theta \in [0, 1]$, is irrelevant to determine the equilibrium allocation as long as (15) does not violate, because both government bonds and private assets are traded as identical securities by type 2 agents in the model.[33] [34]

Unlike the previous case of $A = 0$, the feasible equilibrium allocation set can be expanded or restricted by the level of the asset price, ψ, in (24). For example, if the real interest rates in the channel system, r^c_b, in (18) and the real interest rates in the floor system with LER, r^f_b, in (23) increase, then the asset price falls in (11), so the feasibility condition (24) shrinks toward the origin. If the central bank implement monetary policy in the channel system with open-market operations, $\delta \in [\tilde{\delta}, \rho]$, at zero interest on reserves, $R^m = 1$, then the feasibility condition (24) can be described

[32] the total supply of assets in this economy is sufficiently small as $V + \psi^f < x^*$, ψ^f is defined as $\psi^f = \frac{\beta y}{1-\beta}$.

[33] The only difference between two assets is that the aggregate supply of private assets can be changed by the asset prices, while the real quantity of government bonds is fixed.

[34] In order to maintain this feature, I assume that when $\delta(V + \psi A) > V$ holds from (14)-(15), the central bank will choose $\theta \in [\bar{\theta}, 1]$ where $\bar{\theta} = \delta(1 + \frac{V}{\frac{\beta y u^{'}(x_2)}{1-\beta u^{'}(x_2)}A}) - 1$ is derived from (13)-(14) with $qb = 0$.

as $IC(V+\psi)$ curve in Figure 5a. Note that the slope of $IC(V+\psi)$ curve (24) is less steep than the slope of $IC(V)$ curve (19), because when δ increases, the real interest rate goes down.

It is not simple in the case of floor system with LER, because the central bank can use either the interest on reserves(R^m) or open market operation(δ) and also the effect on the real interest rate can vary by these policy tools. Therefore, we consider the shape of the feasibility curve (24) by considering the effect of each policy variable, R^m or δ, on the real interest rates.

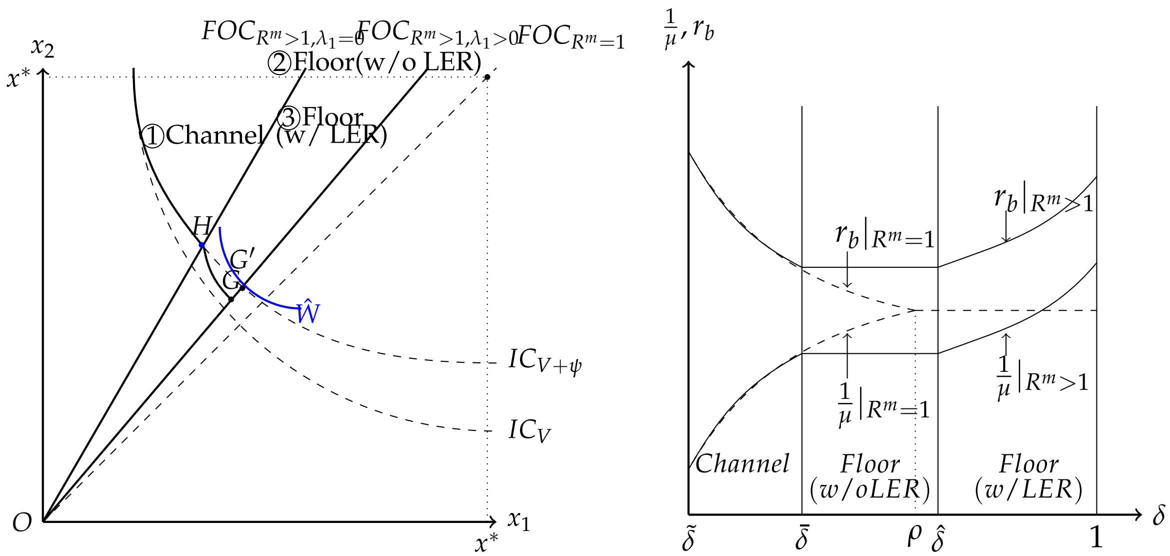
5.1 Open-market Operations($\triangle\delta$)

In Figure 3, given $R^m \in (1, \frac{1}{\tilde{q}})$, as the central bank raises δ from $\tilde{\delta}$ to 1, the equilibrium allocation starts with the channel system regime and passes the floor system without LER regime and reaches floor system with LER regime. Since the open-market operations are ineffective in the floor without LER regime, in Figure 5a once the equilibrium allocation arrives at the point H with $\delta = \tilde{\tilde{\delta}}$, it remains when δ increases further. When δ reaches $\hat{\delta}$ in the floor system with LER regime, raising δ tightens the binding incentive constraint (21), so the real interest rate goes up and the asset price falls. Therefore, we have a kink at the point H and the feasibility of equilibrium allocations in the floor system with LER becomes restricted more than that in the channel system.

Corollary 1. *When δ is raised, the feasible set of equilibrium allocation is reduced further in the floor system with LER than in the channel system.*

Proof. *See the appendix.*

As summarized in Table 1, when δ increases, both the inflation rate and the real interest rate decrease in the channel system whereas the inflation rate falls and the real interest rate goes up in the floor system with LER, so we can describe the inflation rate and the rate of return on government bonds as shown in Figure 5b. The dotted lines in Figure 5b show the paths of the rates of return on currency/CBDC and government bonds when δ increases only in the channel system.



(a) Equilibrium with Private Assets

(b) Rates of return on currency/CBDC and bonds

[Figure 5. Open-market Operations($\triangle\delta$)]

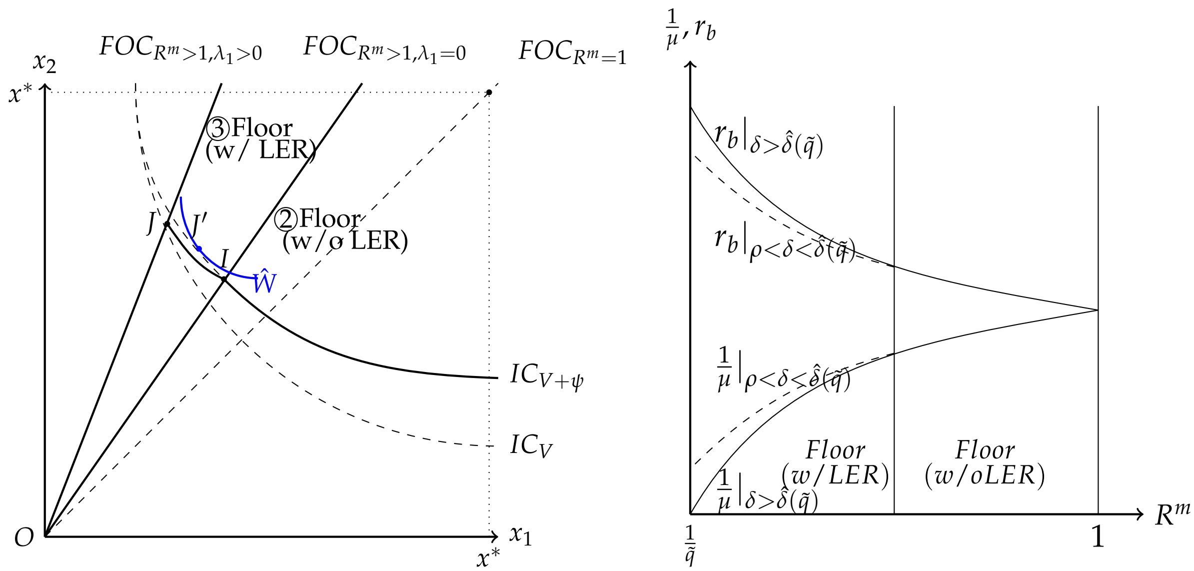
5.2 Interest on Reserves($\triangle R^m$)

Similarly, given $\delta \in (\rho, 1)$ when the central bank raises R^m from 1 to $\frac{1}{\bar{q}}$, the equilibrium allocation starts with the floor system without LER and reaches the floor system with LER as shown in Figure 3. Since changing the interest on reserves in the floor system without LER has the same effect as in the channel system, when R^m is raised, the equilibrium allocation moves along the curve (24) from point I to point J in Figure 6a. However, when R^m increases, the real interest rate in the floor system with LER goes up further than one in the floor system without LER as shown in Proposition 3. Thus, the asset prices go down further and the feasible allocation set shrinks as described in Figure 6a.

Corollary 2. *When the interest on reserves, R^m, is raised, the feasible set of equilibrium allocation is reduced further in the floor system with LER than in the floor system without LER.*

Proof. *See the appendix.*

Given a sufficiently large excess reserve, δ, if R^m is raised then the allocation moves from the floor system without LER into the floor system with LER in Figure 3. In this case both the inflation rate and the real interest rate increase either in the floor system without LER or in the floor system with LER. However, both the inflation rate and the real interest rate rise further in the floor system with LER as described in Figure 6b.



(a) Equilibrium with Private Assets (b) Rates of return on currency and bonds

[Figure 6. Interest on Reserves($\triangle R^m$)]

5.3 Welfare

Since the utility and production functions are linear in the CM, all the surplus is generated from trades in the DM. So the welfare function is

$$W(x_1, x_2) = \rho\{u(x_1) - x_1\} + (1-\rho)\{u(x_2) - x_2\} + yA, \tag{30}$$

which consists of the trading gains in the DM and the dividend from the Lucas tree. However, it is important to consider the costs of operating a currency/CBDC system when we evaluate the effects of monetary policy. It includes not only the direct cost of managing the stock of currency and data in the central bank server, but also the indirect social costs to inhibit the illegal activities such as theft, counterfeiting, hacking, etc. One way to reflect this operating cost is to assume that a fraction ω of currency/CBDC trade is socially useless as suggested in Williamson (2012, 2016). Then, our welfare measure becomes

$$\hat{W}(x_1, x_2) = \rho\{(1-\omega)u(x_1) - x_1\} + (1-\rho)\{u(x_2) - x_2\} + yA. \tag{31}$$

As shown in Figures 5a and 6a, raising δ and R^m in the floor system with LER regime shrinks the set of the feasible equilibrium allocations. So, if given (δ, R^m) an equilibrium allocation is located at the floor system with LER in Figure 3, there is a possibility to improve welfare by expanding the feasibility set. For example, by reducing δ to $\bar{\delta}$, and then reducing R^m in the floor system without LER regime, the allocation can move from point G to point H, and then to point G' in Figure 5a, where $\hat{W}$ represents the welfare curve. Similarly, by reducing R^m to reach the floor system without LER regime, and then reducing δ to the channel system in Figure 3, the allocation can move from point J to point I, and then to point J' in Figure 6a. If there are no restrictions on the policy variables (δ, R^m), the channel system with $\delta \in (\tilde{\delta}, \rho]$ and $R^m = 1$ is desirable to maximize the feasibility of the equilibrium allocations.

Proposition 4. *If ω is sufficiently large, then the equilibrium allocations in the floor system with LER are suboptimal.*

Proof. *See the appendix.*

Proposition 4 shows that reducing the amount of excess reserves can be beneficial when the cost of operating a currency/CBDC system is sufficiently large. If the cost of operating a currency/CBDC system is large, then it is optimal to raise the nominal interest rate high enough because it can discourage the currency trade. When the nominal interest rate goes up, the aggregate liquidity supply decreases further with the large excess reserves, thus a policy mix that raises the interest on reserves and reduces the excess reserves simultaneously can improve welfare.

5.4 Discussion

We find out that when CBDC is introduced with the large excess reserves, the real interest rate can go up sharply as the level of the interest on reserves is raised. This result is not desirable from the welfare perspective, because it can reduce the aggregate liquidity supply excessively in an economy. Allowing to access the reserve accounts provides an opportunity for agents to convert the illiquid asset, reserves, into the liquid one, CBDC, at any time. It can reduce the total supply of illiquid assets when the large excess reserves are held by banks, thus the liquidity distribution across the agents can be inefficient, and the more liquidity supply is required. Therefore, if the large amount of excess reserves is inevitable, there could be an alternative solution: Instead of liquid excess reserves, the central bank can issue illiquid bonds such as the central bank's bill or ON-RRP.[35] Issuing the illiquid debt can prevent the banks from holding liquid assets excessively in their portfolio, which can be used as CBDC at any time.

6 Conclusion

In this paper we study the effect of introducing CBDC on monetary policy tools in the floor system with the large excess reserves. In the model the banks provide a liquidity insurance to the agents

[35] Overnight reverse repurchase agreement(ON-RRP) facility sells a security to an eligible counterparty and simultaneously agrees to buy the security back the next day.

who are exposed to the idiosyncratic liquidity risk. Since the large excess reserves can inhibit the banks from separating the types by liquidity needs, there could exist a new regime in the floor system where open-market operations can be effective with a return dominance. When we consider the aggregate liquidity effect with private assets, the set of feasible equilibrium allocations can be reduced in this regime. In this case we can expand the feasibility by reducing either the interest on reserves or the proportion of excess reserves in the bank's balance sheet or both.

This paper takes a step forward to understand monetary policy with the large excess reserves when CBDC is introduced. It provides a theoretical model in which we can implement both the interest on reserves and open-market operations in the floor system. However, it also leaves some questions unanswered. For example, this model uses an insurance contract with truth-telling constraints instead of secondary financial markets under private information. In a respect of convertibility we may ask how the secondary market friction can also influence the effectiveness of these monetary policy tools. Moreover, since the truth-telling constraints can prevent bank runs in the model, we may further study how the fragility of banks can be associated with the introduction of CBDC. Finally, there are other respects of excess reserves which could be considered in future research. For example, the excess reserves can be very helpful for banks when they are exposed to an aggregate liquidity shock.

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Appendix

A Proofs

Lemma 1. *In channel systems the truth-telling constraints (6)-(7) do not bind.*

Proof. *Given the utility function,* $u(x) = \frac{x^{1-\gamma}}{1-\gamma}$ *where* $\gamma < 1$*, the first-order conditions (17)-(18) can be rewritten as* $x_2 = \frac{1}{q}^{\frac{1}{\gamma}} x_1$*. Since* $\frac{1}{q} > R^m \geq 1$ *in channel systems,* $x_2 = \frac{1}{q}^{\frac{1}{\gamma}} x_1 > R^m x_1$ *always holds, so the truth-telling constraint (7) does not bind in equilibrium. The truth-telling constraint for type 1 buyers (6) does not bind in channel systems because there are no excess reserves to use,* $x_2^m = 0$*. QED*

Lemma 2. *In the floor system the truth-telling constraint (7) never binds, but the truth-telling constraint (6) can bind when the excess reserves are sufficiently large as* $\delta > \hat{\delta}$ *where* $\hat{\delta}$ *satisfies* $\frac{\hat{\delta}-\bar{\delta}}{1-\bar{\delta}} = (R^m)^{1-\frac{1}{\gamma}}$*.*

Proof. *When* $\lambda_1 = \lambda_2 = 0$*, the equilibrium conditions (17)-(19) still hold given* $\frac{1}{q} = R^m > 1$*, so we have* $x_1 = (R^m)^{-\frac{1}{\gamma}} x_2$ *in equilibrium. The truth-telling constraint for type 2 buyers (7) does not bind because* $\frac{1}{q}^{\frac{1}{\gamma}} > R^m$ *at* $\frac{1}{q} = R^m > 1$*. However, the truth-telling constraint for type 1 buyers (6) can bind when the excess reserves are sufficiently large. Given* $R^m > 1$ *there exists a threshold* $\hat{\alpha} \in (0,1)$ *which satisfies with* $\hat{\alpha} = (R^m)^{\frac{1-\gamma}{\gamma}}$ *from (21) and the first-order condition,* $x_1 = (R^m)^{-\frac{1}{\gamma}} x_2$*. Since* $\hat{\alpha}$ *is associated with* $\hat{\delta}$ *as* $\hat{\alpha} = \frac{\hat{\delta}-\bar{\delta}}{1-\bar{\delta}}$*, we know that (21) binds at* $\delta \in (\hat{\delta}, 1]$*, which is equivalent with* $\alpha \in (\hat{\alpha}, 1]$*. Note that* $\hat{\delta} > \bar{\delta}$ *because (21) does not bind at* $\alpha = 0$*. QED*

Proposition 1. *Given* R^m*, the inflation rate decreases in* δ *while the real interest rate on government bonds increases in* δ *in the floor system with LER.*

Proof. *Given* R^m*, if* δ *decreases, then the equilibrium allocation* (x_1^f, x_2^f) *moves toward point C in Figure 4. We can check the changes in the inflation rate and the real interest rate by reducing* x_1^f *along the curve (19) given the same* R^m*.*

$$\begin{aligned} \frac{\partial \mu}{\partial x_1^f} &= \beta\{\rho u''(x_1^f) + (1-\rho) R^m u''(x_2^f) \frac{\partial x_2^f}{\partial x_1^f}|_V\} = \beta\rho\gamma u'(x_1^f)\{-\frac{1}{x_1^f} + \frac{R^m}{x_2^f}\} < 0, \\ \frac{\partial q\mu}{\partial x_1^f} &= \beta u''(x_2^f) \frac{\partial x_2^f}{\partial x_1^f}|_V > 0. \end{aligned} \tag{A.1}$$

In (A.1) we use $\frac{\partial x_2^f}{\partial x_1^f}|_V = -\frac{\rho u'(x_1)}{(1-\rho) u'(x_2)}$ *from (19), given* $-\frac{x u''(x)}{u'(x)} = \gamma$*. Thus, the inflation rate goes up while the real interest rate on government bonds decreases when* x_1^f *decreases by reducing* δ*. QED*

Proposition 2. *Given* δ*, both the inflation rate and the real interest rate on government bonds increase in the interest of reserves,* R^m*, in the floor system with LER.*

Proof. *By plugging (19) and (21) into (22)-(23), we can have*

$$\begin{aligned} \frac{\mu^f}{\beta} &= \rho u'(x_1^f) + (1-\rho)R^m u'(x_2^f) = (1-\alpha)\rho u'(x_1^f) + \frac{\alpha V}{x_1^f}, \\ \frac{1}{\beta r_b^f} &= u'(x_2^f). \end{aligned} \tag{A.2}$$

Given δ, if R^m is raised then x_1^f decreases and x_2^f increases along the IC curve (19), so both μ^f and r_b^f increase in (A.2). QED

Proposition 3. *At the same level of δ, when the interest on reserves is raised, the inflation rate increases less and the real interest rate on the government bonds decreases more in the floor system with LER compared to the floor system without LER.*

Proof. *By plugging (17)-(18) and (21) into (19), for each $i = \{c, f\}$, we can have*

$$\begin{aligned} &x_1^c u'(x_1^c)\{\rho + (1-\rho)(R^m)^{\frac{1-\gamma}{\gamma}}\} = V, \\ &x_1^f u'(x_1^f)\left[\rho + (1-\rho)\alpha^{\gamma-1}(R^m)^{1-\gamma}\right] = V. \end{aligned} \tag{A.3}$$

By using (A.3) we can rewrite (17) and (22) as

$$\begin{aligned} \frac{\mu^c}{\beta} &= \left(\frac{\rho+(1-\rho)(R^m)^{\frac{1-\gamma}{\gamma}}}{V}\right)^{\frac{\gamma}{1-\gamma}}, \\ \frac{\mu^f}{\beta} &= \left(\frac{\rho+(1-\rho)\alpha^{\gamma-1}(R^m)^{1-\gamma}}{V}\right)^{\frac{\gamma}{1-\gamma}}\{\rho + (1-\rho)\alpha^{\gamma}(R^m)^{1-\gamma}\}, \end{aligned} \tag{A.4}$$

respectively. From (A.4) we can have

$$\frac{\partial \mu^f}{\partial R^m} = \frac{\partial \mu^c}{\partial R^m}\left(\rho\alpha_\kappa^{-\gamma}(R^m)^{1-\gamma} + (1-\rho)\right)^{\frac{\gamma}{1-\gamma}}\left[\gamma + (1-\gamma)\frac{\rho\alpha + (1-\rho)\alpha^{\gamma}(R^m)^{1-\gamma}}{\rho + (1-\rho)\alpha^{\gamma}(R^m)^{1-\gamma}}\right]. \tag{A.5}$$

Given the same level of R^m, since $x_1^c < x_1^f$, $(R^m)^{\frac{1-\gamma}{\gamma}} > \alpha^{\gamma-1}(R^m)^{1-\gamma}$ holds in (A.3).[36] *By using this inequality, we can find out that $|\frac{\partial \mu^f}{\partial R^m}| < |\frac{\partial \mu^c}{\partial R^m}|$ holds in (A.5). Since the Fisher equation holds in the model, the effect on the real interest rate in the floor system with LER is greater than one in the floor system without LER. QED*

Corollary 1. *When δ is raised, the feasible set of equilibrium allocation is reduced further in the floor system with LER than in the channel system.*

Proof. *By Proposition 2, the real interest rate on private assets goes up when δ increases in the floor system with LER. Therefore, the asset price decreases and the feasibility condition moves toward the origin. QED*

Corollary 2. *When the interest on reserves, R^m, is raised, the feasible set of equilibrium allocation is reduced further in the floor system with LER than in the floor system without LER.*

[36] This inequality, $(R^m)^{-\frac{1}{\gamma}} < \frac{\alpha}{R^m}$, implies that the slope of the first-order condition (17)-(18) in the channel system(or the floor system without LER) is greater than the slope of the first-order condition (21) in the floor system with LER in Figure 4.

Proof. *By Proposition 3, when the interest on reserves, R^m, increases, the real interest rate on private assets goes up further in the floor system with LER comparing to the floor system without LER. Therefore, the asset price decreases further and the feasible set of the equilibrium allocation is more restricted. QED*

Proposition 4. *If ω is sufficiently large, then the equilibrium allocations in the floor system with LER are suboptimal.*

Proof. *For given a monetary policy (R_0^m, δ_0) in the floor system with LER, we can find out a policy set (R_1^m, δ_0) which is located on the borderline between the floor system without LER and the floor system with LER in Figure 3. Let $(x_1, x_2) = (\hat{x}_1, \hat{x}_2)$ solve (21) and (24) with $\psi = \frac{\beta y u'(x_2)}{1-\beta u'(x_2)}$ at $R^m = R_1^m$. The slopes of the adjusted welfare function (26) and the IC curve (24) at this allocation $(\hat{x}_1, \hat{x}_2)$ are*

$$\frac{\partial x_2}{\partial x_1} |_W = -\frac{\rho\{(1-\omega)u'(\hat{x}_1) - 1\}}{(1-\rho)\{u'(\hat{x}_2) - 1\}} \tag{A.6}$$

and

$$\frac{\partial x_2}{\partial x_1} |_V = -\frac{\rho(1-\gamma)u'(\hat{x}_1)}{(1-\rho)(1-\gamma)u'(\hat{x}_2) - K'(\hat{x}_2)}, \tag{A.7}$$

respectively, where $K(x_2) := \frac{\beta y A u'(x_2)}{1-\beta u'(x_2)}$ and $K'(x_2) < 0$. Note that there exists a threshold $\omega^*(R_1^m) \in (0,1)$ at which the two slopes are equal. If $\omega > \omega^*(R_1^m)$, then both x_1 and x_2 can increase by choosing the original $R^m = R_0^m$ and reducing $\delta < \delta_0$. QED

B Equilibrium allocations without binding incentive constraints

B.1 Channel Systems

In a channel system the central bank can set the interests on CBDC and reserves, R^d and $R^d R^m$, lower than the nominal interest rate on government bonds, $\frac{1}{q} - 1$, which is controlled by open-market-operations, δ. So banks will not hold any excess reserves in equilibrium, $m = 0$ and $x_2^m = 0$, in this case. We also know that the constraints (3)-(5) always bind when the first-best allocation is not feasible. The first step of characterization is to solve for the equilibrium conditions without the binding incentive constraints (6)-(7). If $\lambda_1 = \lambda_2 = 0$, then the first-order conditions (9) and (11) can be reduced into

$$\frac{\mu}{\beta R^d} = u'(x_1), \tag{B.1}$$

$$q\frac{\mu}{\beta} = u'(x_2), \tag{B.2}$$

respectively. By using these first-order conditions (18)-(19) and the binding constraints (3) and (5), the government budget constraint (13) with $A = 0$ can be transformed into a feasibility condition,

$$\rho x_1 u'(x_1) + (1-\rho) x_2 u'(x_2) = V. \tag{B.3}$$

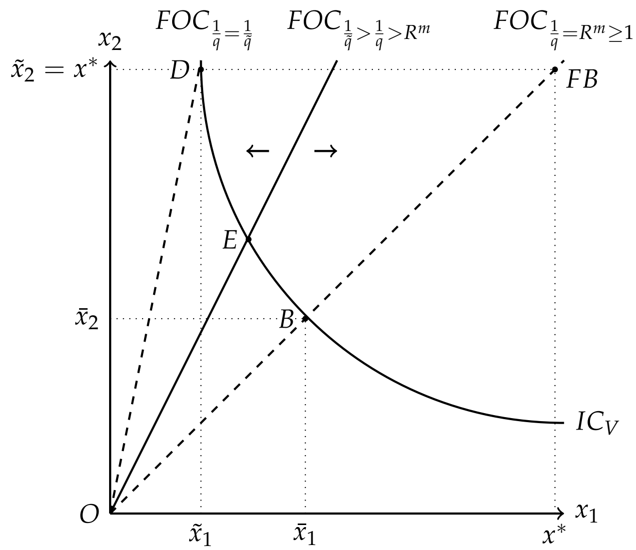
Note that given a nominal interest rate target $\frac{1}{q} - 1$, x_1 and x_2 have a strictly positive relationship as $qR^d u'(x_1) = u'(x_2)$ in (17)-(18) and a strictly negative relationship in (19). So there is a unique equilibrium allocation (x_1, x_2), which satisfies with (17)-(19): The point E in Figure 2 describes the equilibrium allocation (x_1, x_2) that intersects FOC curve from (17)-(18) and IC curve (19). In order to support this equilibrium allocation (x_1, x_2, q), the central bank can implement open-market-operations by choosing δ as

$$\delta = \frac{\rho x_1 u'(x_1)}{V}, \tag{B.4}$$

which can be derived from (14) and (19). Notice that the equilibrium allocation (x_1, x_2) is determined by the relative rate between the interest on CBDC, R^d, and the nominal interest rate, $\frac{1}{q}$. That means, when R^d varies, only the nominal interest rate and the inflation rate, μ, will change along with it and the other real allocations are maintained. Thus, without the loss of generality, we assume that the interest on CBDC is normalized as one, $R^d = 1$, from now on.

In Figure 2, given $R^m \geq 1$ the equilibrium allocations (x_1, x_2) are feasible on the IC curve between the points B and D associated with $\frac{1}{q} \in [R^m, \frac{1}{\tilde{q}}]$ where $\tilde{q}$ is defined as the lower bound of q at the point D. Since δ is increasing in x_1 in (20), each $q \in [\tilde{q}, \frac{1}{R^m}]$ is implemented by a corresponding $\delta \in [\tilde{\delta}, \bar{\delta}]$, where $\tilde{\delta}$ and $\bar{\delta}$ are the lower and upper bound of δ: Given the utility function, $u(x) = \frac{x^{1-\gamma}}{1-\gamma}$, $q = \left(\frac{1-\rho}{\rho}\frac{\delta}{1-\delta}\right)^{\frac{\gamma}{1-\gamma}}$ can derived from (17)-(20). As shown in Figure 2, when $(\tilde{x}_1, \tilde{x}_2)$ and $(\bar{x}_1, \bar{x}_2)$ are defined as the allocations at $q = \tilde{q}$ and $q = \frac{1}{R^m}$, respectively, each $\tilde{\delta}$ and $\bar{\delta}$ satisfies (20) with $\tilde{x}_1$ and $\bar{x}_1$, respectively.[37]

[37] Note that in case of the lower bound, $\frac{1}{q} = R^m$, $\delta > \bar{\delta}$ can also support the same equilibrium allocation with $\delta = \bar{\delta}$, because the excess reserves can be held at $\frac{1}{q} = R^m$.



[Figure 2. Equilibrium in Channel System]

Lemma 3. *In channel systems the incentive constraints (6)-(7) do not bind.*

Proof. *See the appendix.*

Lemma 1 shows that both the incentive constraints do not bind in the channel system. The insurance contract provides currency for type 1 buyers and government bonds for type 2 buyers to make the marginal rate of substitution between currency and government bonds equivalent to the given nominal interest rate. Without the excess reserves, type 1 buyers prefer currency/CBDC because it is unavailable to use government bonds in retail transactions. Type 2 buyers prefer government bonds as long as the rate of return in government bonds is greater than the rates of return on currency and CBDC. Therefore, the optimal insurance contract does not violate the truth-telling incentive constraints without excess reserves.

In a channel system, monetary policy is implemented only by open-market operations, because the level of the interest rate on reserves, R^m, is irrelevant to determine the equilibrium allocations. For example, suppose that the central bank tries to raise the nominal interest rate, $\frac{1}{q} - 1$, by reducing δ, i.e. open-market sales. Then, the currency is absorbed and the government bonds are injected into the market, so the liquidity premium on currency goes up whereas the liquidity premium on the government bonds falls. Thus, the consumption in the currency trade, x_1, falls while the consumption in the collateral transaction, x_2, rises.[38] Since the real rate of return on currency decreases, the inflation rate rises. Meanwhile, the real rate of return on government bonds increases. Therefore, the nominal interest rate also rises by the Fisher equation.[39]

[38]In Figure 2, the *IC* curve remains, while the *FOC* curve shifts to the left.
[39]This mechanism is the same as described in Williamson (2012,2016).

In channel system, open-market operations can be ineffective only at the lower bound, $\frac{1}{q} = R^m$. [40] Since the rates of return on reserves and government bonds are equal at $\frac{1}{q} = R^m$, we can have excess reserves, $m \geq 0$ and $x_2^m \geq 0$, in this case. The allocation (x_1, x_2) is determined from (17)-(18) with $\frac{1}{q} = R^m$, but reserves and government bonds are indeterminate in equilibrium from (13)-(14) and (19)-(20), $m + b = V - \rho x_1 u^{'}(x_1)$. Thus, if $\delta > \bar{\delta}$, the OMOs is no longer effective.

In sum, if the monetary policy is implemented in channel system, there is no effect on real allocations when an account-based CBDC is introduced to the agents.

B.2 Floor System without Large Excess Reserves

When $\lambda_1 = 0$, the equilibrium allocation at $R^m > 1$ in a floor system is the same as one in a channel system with $\frac{1}{q} = R^m > 1$, because given $\lambda_1 = 0$, $\frac{1}{q} = R^m$ holds in (10)-(11) and the equilibrium conditions (17)-(19) still hold. In this case reserves are used as a perfect substitute for government bonds and the rates of return on both assets are equal. As shown in the lower bound case in the channel system, $R^m = 1$, OMOs are no longer effective because reserves and government bonds are indeterminate in equilibrium as $m + b = R^m\{V - \rho x_1 u^{'}(x_1)\}$ from (13)-(14) and (19)-(20). However, the nominal interest rate target, $\frac{1}{q} - 1$, can be always achieved by adjusting the level of the interest rate on reserves, R^m. So, the interest on reserves is effective as a policy tool and we can obtain the same allocation by raising R^m in the floor system instead of reducing δ in the channel system. Therefore, the impacts of monetary policy in both systems on the real interest rate and/or the inflation rate are equivalent.

Note that although the equilibrium allocations are identical, the implementation mechanism in the floor system is different from the one in the channel system. In the channel system, the real quantities of currency/CBDC and government bonds pin down the rates of return on currency/CBDC and government bonds, whereas in the floor system the relative price between currency/CBDC and reserves, that is, the interest rate on reserves, determines the demands for currency/CBDC and reserves, respectively.

[40] In this respect the equilibrium feature at the lower bound is similar to that in the floor system.