Interest on Reserves, Interbank Insurance and Monetary Policy*

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December 15, 2019

Abstract

This paper examines the effectiveness of monetary policy when interest is paid on reserves by considering the banking sector’s insurance incentives. An asset-exchange model is constructed with idiosyncratic liquidity risk, where one type of banks require currency to trade while the other banks can use any assets including reserves and government bonds as collateral. There arises an interbank insurance to distribute assets efficiently by types. When the interest on reserves provides an opportunity for banks to convert currency and reserves each other, large amount of excess reserves can make it difficult to reveal the types under private information. If government bonds is more costly to convert into currency than reserves, open-market operations can be effective even with the large excess reserves. However, the monetary policy with large excess reserves can be sub-optimal because the liquidity is distributed inefficiently. Key Words: excess reserves, truth-telling constraints, liquidity trap, market segmentation JEL Codes: E42,E44,E52.

*An earlier version of this paper has been circulated under the title "Interest on Reserves, Banking Contracts and Monetary Policy". I especially thank Stephen Williamson for his continuous support. I thank Gaetano Antinolfi, Costas Azariadis, Josh Hendrickson, David Van Hoose and Todd Keister for their advice and comments. All errors are my own.

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1 Introduction

In response to the last Great Recession, the Federal Reserve has implemented various types of unconventional monetary policy accompanied with the interest on reserves. Since the large amount of excess reserves, generated by the Federal Reserve’s other lending facilities and asset purchase programs, was exerting downward pressure on the federal funds rate, in October 2008 the Federal Reserve began to pay interest on excess and required reserve balances and shifted the monetary policy framework from channel (or corridor) system, which was used for more than 40 years, to floor system.

This change in monetary policy environment provides a challenge to policy makers because open-market operations are not feasible to adjust the federal funds rate any longer given the large amount of excess reserves. As a policy normalization plan, in September 2014 the Federal Reserve introduced two ways:¹ The first one is a lift-off, which adjusts the interest on reserves to move the federal funds rate into the target. The second approach is unwinding, which reduces the Federal Reserve’s balance sheet size by using overnight reverse repurchase (ON-RRP) agreement facility. Since we have limited experience in this floor system with the large excess reserves, there are some questions that we might ask. For example, between the two policy tools for normalization, lift-off and unwinding, Which one is more efficient? Which one should be executed first and which one should follow the other?

This paper focuses on the optimal behavior of the banking sector to answer these questions because the banking sector’s choice of asset portfolio and the monetary policy interact with each other. For example, open-market operations adjust the supply of reserves by intervening in the federal funds market. Thus, the asset portfolio of the banks, which participate in the federal funds market, is naturally affected by these operations. On the other hand, when the banks choose the asset portfolio to maximize their profits, the demand for reserves can also influence the effectiveness of monetary policy implementation. In this paper I compare the effectiveness of monetary policy in the different regimes by considering the financial intermediaries’ optimal choices on their asset portfolio.

When the financial intermediaries, or eventually their depositors, are exposed to idiosyncratic preference shock on the specific types of assets for liquidity, there arises a coalition of banks endogenously to provide an interbank insurance for efficient liquidity distribution.² For instance, if

¹See Williamson (2015) for more detail.
²See Allen et al. (2009) and Freixas et al. (2011) for more details on the liquidity insurance in the interbank
one type of banks requires only currency for transactions while the other type can use any asset holdings including reserves and government bonds for collateral, then an insurance contract can maximize the \textit{ex ante} welfare by providing currency to the former type while the rest of assets to the latter type. However, if the information about one’s type is private, revealing the types can be critical because one type can mimic the other type under anonymity. Therefore, the capability of transforming one asset to the other is important to prevent the agents from mimicking. For example, suppose that currency and reserves can be converted from one to the other, but it is costly to convert government bonds into currency. Then, less liquid assets such as government bonds are useful to reveal the types because the insurance contract can distinguish the two types by offering different types of assets. However, if the whole banking sector’s balance sheet is filled only with currency and reserves, it is impossible to distinguish the two types. In this respect, although reserves and government bonds can be used in the same type of trade, they are not perfect substitutes when the supply of government bonds is not sufficient to reveal the types. This difference between reserves and government bonds can influence the effectiveness of monetary policy even when excess reserves are held in the banking sector.

In order to study this issue I develop a search-theoretical monetary model a la Lagos and Wright (2005) and Rocheteau and Wright (2005) with an insurance arrangement shown in Sanches and Williamson (2010) and Williamson (2012, 2016). This model has an advantage of incorporating insurance contracts in a simple way and is also highly tractable with an array of assets. In the model agents can produce consumption goods with an elastic labor supply, but cannot consume by themselves. Under limited commitment and lack of record-keeping, agents need a medium of exchange to trade for consumption: one type of agents can use only currency for trade while the other type can use the whole asset portfolio including reserves and government bonds as collateral. The lack of memory assumption not only makes assets essential for trade, but also keeps the type information of the agents secret.\footnote{If record-keeping is available, credit or taxation can help to achieve the optimal equilibrium allocation even with private information.} An insurance contract with truth-telling incentive constraints arises endogenously to reveal the types by offering different types of assets. After receiving the provided assets, each agent has an opportunity to convert currency and reserves each other by accessing to the central bank.\footnote{In equilibrium the agents do not change their asset portfolio, but this opportunity can affect their demands for the assets \textit{ex ante}.} In the model the fiscal authority can collect a lump-sum tax and issue government bonds, but play only a passive role by keeping the real value of outstanding market.
government bonds constant forever. The central bank issues currency and reserves by holding government bonds and can provide a positive interest on reserves in the floor system. In the extended model we introduce a Lucas tree with a fixed supply to consider an aggregate liquidity effect associated with the real interest rates, but the supply of total assets is still scarce for satisfying the whole transaction demands.

In the model the central bank implements the monetary policy by choosing the level of the interest on reserves and the proportion of currency plus reserves in the total asset portfolio of the banks. Given the monetary policy variables we have three equilibrium cases, i.e. channel system, floor system with liquidity trap, and floor system without liquidity trap, where the liquidity trap implies that open market operations, i.e. the exchange between reserves and government bonds, are no longer effective in the real economy.

In the channel system without excess reserves, the monetary policy adjusts the supply of assets through open market operations to control the liquidity premium of each asset separately in each type of trades. In the floor system with excess reserves where the truth-telling incentive constraints do not bind, the rates of return on currency and government bonds are determined by the interest on reserves. Since both reserves and government bonds are used for the same type of transaction, there is no return dominance between reserves and government bonds and open market operations are no longer effective. However, adjusting the interest rate on reserves can make the same effect as open-market operations in the channel system.

When the excess reserves are sufficiently large, one of the truth-telling incentive constraints can bind. In this case there exists a new regime, the floor system without liquidity trap, under which open-market operations are effective. There are two effects with the binding incentive constraint. One is the liquidity supply effect: For example, open-market purchases, i.e. injecting reserves and absorbing government bonds, make it more difficult to separate the types. Thus, the insurance contract provides less reserves and government bonds to the collateral trading agents while more currency to the currency trading agents. The other effect is that both currency and collateral trading markets can be related through the binding incentive constraint, because the liquidity premium of one asset depends on not only the corresponding trading market, but also the other type of trading market condition. For instance, the government bonds is useful for the collateral trade, but also helpful for the currency transaction by relaxing the truth-telling incentive constraint.

With this assumption, we can focus on the monetary policy separated from the fiscal policy. Moreover, it is one simple way to consider the scarcity of total asset supply in an economy.
It is found that the liquidity premia of currency and government bonds can move into the same direction by an open market operation.

This result provides an implication in monetary policy implementations. In the model an open-market operation can affect the real interest rates by shifting the liquidity from one type of trade to the other. However, given that the interest on reserves provides an opportunity to convert assets easily, an open-market operation with the large excess reserves can be less successful because the liquidity premium on one asset reflects the both trading market conditions, even though the markets are segregated by the types of assets which can be accepted. In the extension with private assets where monetary policy has an impact on the aggregate liquidity supply, the channel system is preferred to the floor system without liquidity trap in a perspective of feasibility. Since the open market purchases with the binding incentive constraint leads the real interest rate to rise more, the aggregate liquidity can be reduced further in the floor system without liquidity trap.

1.1 Related Literature

There is a growing literature on monetary policy with the interest on reserves. Since the interest on reserves can be used as a policy tool independently, Goodfriend (2002), Ennis and Weinberg (2007), and Keister et al. (2008) point out that the central bank can have an additional degree of freedom to choose the quantity of reserves by providing the interest on reserves.\(^6\) Recently, Ireland (2014) and Ennis (2018) pay attention to the decoupling between the supply of reserves and the price level given the interest on reserves: Ireland (2014) finds out that the quantity of reserves can affect the nominal variables when there exists a cost for managing reserves. Similarly, Ennis (2018) shows that the link between the supply of reserves and prices can be tightened again when the bank capital constraint binds with a large balance sheet. In my model the price variables are determined by the government budget constraint rather than the supply of reserves, which is similar to Cochrane (2014).\(^7\) However, the supply of reserves can be used as a policy variable because the interest on reserves provides an opportunity to deposit currency and/or withdraw from reserves.\(^8\)

\(^6\)In this respect Kashyap and Stein (2012) argue that the interest on reserves can be useful for aiming another target such as financial stability.

\(^7\)Cochrane (2014) shows that when fiscal policy is restricted, the inflation rate can be determined by the government budget constraint although reserves and government bonds are perfect substitutes.

\(^8\)In this paper we also have a case of a liquidity trap, where open-market-operations are irrelevant, given the interest on reserves. This liquidity trap is close to the one in Wallace (1981) rather than Williamson (2012) and Rocheteau et al. (2018), because excess reserves and government bonds can be used for the same type of transactions in the model. In Williamson (2012) and Rocheteau et al. (2018), the liquidity trap arises when the rates of return on money and government bonds are equal because the supplies of both assets are scarce.
previous literature. In the model when the truth-telling incentive constraint binds with a sufficiently large excess reserves, monetary policy is less effective. This result is similar to Agenor and Aynaoui (2010) and Bech and Klee (2011), but the mechanism is different from their models. Agenor and Aynaoui (2010) show that given a precautionary demand for excess reserves, contractionary monetary policy can be less effective with prevailing excess reserves. Bech and Klee (2011) build a limited participation model with Over-the-Counter market to show that the federal funds rate rises less when the interest on reserves increases. In Armenter and Lester (2017) the interest rate spread in a corridor system can have a real effect because the spread between the two rates determines the trading gain and the efficiency of matching in their model. In my model the interest rate spread in a floor system can be effective when the truth-telling incentive constraint binds, because it is associated with the liquidation cost of illiquid assets.

This paper is also related to the literature that studies the bank’s optimal decision with the interest on reserves. Dressler and Kersting (2015) and Martin et al. (2016) focus on the bank’s lending behavior given the excess reserves, while Dutkowsky and VanHoose (2013) and Dutkowsky and VanHoose (2017) study the bank’s decision on sweeping and the size of bank balance sheet given the interest on reserves. In this paper, banks provide a liquidity insurance given idiosyncratic liquidity risk as shown in Williamson (2016, 2018), but I focus more on how interest on reserves can affect the bank’s optimal decision under private information and the effectiveness of monetary policy.9

For the welfare evaluation between channel system and floor system, Berentsen et al. (2014) show that there is a welfare improvement in channel system when lending is costly, because banks will hold more reserves ex ante, which can internalize the pecuniary externality in holding reserves. Williamson (2016) shows that the welfare can improve in floor system by injecting more reserves given interest on reserves, when reserves are more useful than government bonds in a liquidity aspect. In this paper, the channel system without excess reserves is preferred to the floor system in a perspective of aggregate liquidity supply, because the monetary policy is more effective when the trading markets are sufficiently segregated.

Finally, this paper is related to some papers that study the liquidity premium of the illiquid assets. Herrenbrueck and Geromichalos (2017) show that the price of illiquid assets has a liquidity premium if the illiquid assets can be traded to obtain liquid assets. In this paper when the interest on

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9Williamson (2018) shows that the effect of raising the interest on reserves is different from the effect of reducing the central bank balance sheet by constructing two-sector banking model with interbank markets.
reserves is implemented, the liquidity premium of the reserves decreases and the liquidity premium in the government bonds goes up, because the less liquid government bonds are more useful for revealing private information as the reserves are more easily convertible to currency. Therefore, when the truth-telling incentive constraint binds, injecting illiquid assets can be beneficial.\textsuperscript{10}

2 Model

The basic model structure is based on Rocheteau and Wright (2005) in which \textit{ex ante} heterogeneous agents trade in bilateral meetings and rebalance their portfolios in the centralized market. Time is discrete over infinite horizon and each period is divided into two sub-periods - the Centralized Market (CM) followed by the Decentralized Market (DM). There is a continuum of buyers and sellers, each with unit mass. An individual buyer has preferences

\[ E_0 \sum_{t=0}^{\infty} \beta^t [-H_t + u(x_t)] \]

where \( H_t \in \mathbb{R} \) is labor supply of the buyer in the CM, \( x_t \in \mathbb{R}_+ \) is consumption of the buyer in the DM, and \( 0 < \beta < 1 \). Assume that \( u(\cdot) \) is strictly increasing, strictly concave, and twice continuously differentiable with \( u'(0) = \infty \), \( u(0) = 0 \), and \( -x \frac{u''(x)}{u'(x)} = \gamma < 1 \) for all \( x > 0 \).\textsuperscript{11} Each seller has preferences as

\[ E_0 \sum_{t=0}^{\infty} \beta^t [X_t - h_t] \]

where \( X_t \in \mathbb{R} \) is consumption of the seller in the CM, and \( h_t \in \mathbb{R}_+ \) is labor supply of the seller in the DM. All the agents can consume and produce in the CM. In the DM buyers can consume, but cannot produce whereas sellers can produce, but cannot consume. One unit of labor inputs can produce one unit of perishable consumption goods in either the CM or the DM.

In the CM all agents meet together and then production and consumption occur. Buyers receive a lump-sum transfer from the government, and the share holders of the Lucas trees receive the realized dividends. The previous debts are paid off, and assets and consumption goods are traded in a Walrasian market. After this settlement and asset-trading stage, each buyer can engage

\textsuperscript{10}For more on the social benefit of illiquid assets, see Kocherlakota (2003) and Shi (2008). Kocherlakota (2003) shows that illiquid assets are beneficial because agents can trade liquid and illiquid assets after observing idiosyncratic shock. Shi (2008) shows that legal restriction on government bonds improves welfare when low marginal utility type cannot trade with bonds.

\textsuperscript{11}If the coefficient of relative risk aversion is greater than one, the asset demand will be strictly decreasing in the rate of return on the asset in this model. For the most of proofs a specific utility function, \( u(x) = \frac{x^{1-\gamma}}{1-\gamma} \), is used.
at most one bank in an insurance contract.

There are two types of assets in this economy. One is a real asset - a divisible Lucas tree. It is endowed to buyers in the initial period, $t = 0$, $CM$ with a fixed supply of $A$ and pays off $y$ units of consumption goods as a dividend in the $CM$ in every period and trades at the price of $\psi_t$ in terms of goods in the period $t$ $CM$. The other type of assets are liabilities of the fiscal authority and the central bank such as currency, reserves and government bonds. The fiscal authority can issue government bonds and provide nominal interest by collecting lump-sum taxes from buyers. One unit of government bonds trades at price $q_t$ in terms of currency in the period $t$ $CM$ and pays one unit of currency in the period $t + 1$ $CM$. The central bank can provide currency and/or reserves by purchasing government bonds. One unit of reserves is a claim for one unit of currency in the next period and sells at price $z_t$ in terms of currency in the period $t$ $CM$. While reserves are account balances with the central bank, currency is a physical object produced by the central bank. One unit of currency sells at price $\phi_t$ in terms of consumption goods in the period $t$ $CM$ and provides no interest.

Buyers, i.e. individual banks in the interbank market, can access the reserve account in the central bank. They can deposit their currency holdings as reserves and/or withdraw currency from their reserve balances before they move into the $DM$. Buyers can also borrow currency from the central bank by posting government bonds and private assets, but it requires a proportional liquidation cost, $1 - \kappa$.\footnote{In reality, a number of depository institutions in U.S. are permitted to have reserve accounts in the Federal Reserves Bank and to use the discount window facility.} I assume that $1 - \kappa$ is sufficiently large to emphasize the illiquidity of government bonds and private investment in our analysis. In this respect reserves and government bonds are different in the model, because agents can earn currency immediately from reserve balances while it is costly to transform government bonds into currency after the Walrasian market is closed in the $CM$. The other difference between reserves and government bonds is that the price of reserves, $z_t$, can be set by the central bank as a policy variable, while the price of government bonds, $q_t$, is determined in the market.

In the $DM$ each buyer meets each seller randomly and the terms of trade are determined by bargaining in the bilateral meeting. For simplicity, I assume that the buyer makes a take-it-or-leave-it offer to the seller in the meetings. There is no record-keeping technology, so the buyers and the sellers cannot verify the trading history of their partners in the $DM$. Moreover, no one can be forced to work under limited commitment. Thus, recognizable assets are essential for trade in the
$DM$ and all of the trades must be quid pro quo.

In a manner similar to Sanches and Williamson (2010), currency is the only means of payment in some $DM$ transactions while the whole asset portfolio can be used as collateral in the other $DM$ transactions. Suppose that $\rho$ proportion of type 1 buyers will meet a seller who accepts only currency while $1 - \rho$ proportion of type 2 buyers will meet a seller who also accepts the claims for other assets such as reserves and government bonds. At the beginning of the $CM$, buyers do not know which type of match they will be in the $DM$. They learn their types after the Walrasian market is closed and their types are private information. Once they know types, they can meet with at most one insurance bank and the central bank before they move into the $DM$.

Timing is described in Figure 1. In the beginning of the $CM$ government debt holders receive a unit of currency by redeeming a unit of government bonds or reserves. Buyers receive lump-sum transfers (or pay lump-sum taxes). Then all buyers and sellers provide labor and trade assets in a Walrasian market. Buyers deposit consumption goods and/or assets with an insurance bank in exchange for an insurance contract. After the liquidity shock is realized, buyers learn their types and $\rho$ proportion of buyers meet the insurance bank to withdraw currency. Then, the buyers can drop by the central bank to convert their asset portfolio. Finally, In the $DM$ buyers meet sellers randomly in the bilateral meeting and can trade with a take-it-or-leave-it offer.

![Figure 1. Time-line](image)

2.1 Insurance Banks

Given the idiosyncratic liquidity risk, an insurance arrangement arises endogenously to allocate currency and other assets across the types of buyers. Without an insurance contract, type 1 buyers could run out of currency and holding idle government bonds while type 2 buyers could hold low-yielding currency instead of other high-yielding assets. So, the insurance contract can provide an liquidity insurance by allocating currency to type 1 buyers and the rest of assets to type 2 buyers. Note that given perfect competition, a representative insurance bank suggests an optimal insurance contract which maximizes the expected utility of buyers.
Under private information the insurance banks cannot verify the type of individual buyers, so one type of buyers can mimic the other type of buyers. Unlike Sanches and Williamson (2010) and Williamson (2016), we focus on this private information along with the convertibility of the reserves in this paper.\footnote{In their models private information does not matter because the reserves cannot be withdrawn.} After the insurance contract provides different types of assets, buyers can deposit their currency holdings as reserves at the central bank and/or can withdraw their reserve balances for currency. Given the interest rates on reserves type 1 buyers can have an incentive to mimic type 2 buyers and vice versa, according to the amount of currency and reserves provided to type 1 buyers and type 2 buyers, respectively. In order to separate the types, insurance banks will provide a sufficient amount of assets and it can affect the monetary policy by restricting the demand for specific kinds of assets.

In the model all the claims issued by the banks and the central bank are not counterfeitable and any agents can propose this insurance contract and play a role as an insurance bank. I assume that buyers can meet only the insurance bank after their liquidity shock is realized to support the contract with stability.\footnote{If ex post trading among the buyers is allowed then the insurance contract is unraveled and collapsed as shown in Jacklin (1987).} Note that this liquidity insurance equilibrium provides higher welfare than assets-trading market equilibrium.\footnote{Given the utility function, \( u(x) = \frac{1}{1+\gamma} \), if \( \gamma = 1 \), then both equilibrium allocations are equally efficient.}

### 2.2 Government

At \( t = 0 \) government bonds are issued, and then currency and reserves are injected by open-market purchases. The revenue of issuing government bonds, currency and reserves is transferred to buyers. After \( t = 0 \), outstanding currency, reserves and government bonds amounts can be supported by taxes or transfers over time. So the consolidated government budget constraints can be written as

\[
\phi_0(C_0 + z_0 M_0 + q_0 B_0) - \psi_0 \theta_0 A = \tau_0 = V
\]

and

\[
\phi_t \{ C_t - C_{t-1} + z_t M_t - M_{t-1} + q_t B_t - B_{t-1} \} + \{ (\psi_t + y) \theta_{t-1} A - \psi_t \theta_t A \} = \tau_t, \ t = 1, 2, 3, \ldots
\]

where \( C_t, M_t \) and \( B_t \) denote the nominal quantities of currency, reserves and government bonds held by the private sector in the CM at time \( t \), respectively, and \( \theta_t \) denotes a proportion of the
Lucas trees purchased by the central bank. Note that there is no restriction for the central bank to buy and sell the private assets such as a Lucas tree. \( \tau_t \) denotes the real value of the lump-sum transfer from the fiscal authority to each buyer in the CM at period \( t \). I assume that the fiscal authority keeps the total value of the outstanding consolidated government debt as a constant, \( V \), after an exogenously fixed amount, \( \tau_0 \) is transferred at \( t = 0 \). Thus, in every period to maintain the real value of outstanding consolidated government debt, the real term of lump-sum transfer \( \tau_t \) is derived passively from

\[
\tau_t = \left( 1 - \frac{\phi_{t+1}}{\phi_t} \right) V + \frac{\phi_{t+1}}{\phi_t} (z_t - 1)M_t + \frac{\phi_{t+1}}{\phi_t} (q_t - 1)B_t + y\theta_tA, \quad t = 1, 2, 3, \ldots
\]

Note that the lump-sum transfer consists of seigniorage from inflation, real interest payments for government bonds and reserves, and investment earning from the Lucas tree.

3 Maximization Problem

In the model insurance contracts are necessary for liquidity insurance. Under perfect competition a representative bank suggests an insurance contract to maximize the buyer’s ex ante expected utility. In this respect the buyer’s problem is trivial because it is solved by the insurance bank. Moreover, the seller’s problem is also trivial since sellers always accept the buyer’s take-it-or-leave-it offer in equilibrium. Thus, to construct an equilibrium we focus on the insurance bank’s maximization problem and asset market clearing conditions. A representative insurance bank solves the following problem in the CM of period \( t \):

\[
\text{Max} \quad d_t, c_t, m_t, a_t, x_{1t}, x_{2t}^m, x_{2t}^b \quad \text{subject to} \quad d_t - c_t - z_t m_t - q_t b_t - \psi_t(1 - \theta_t)a_t + \\
\left\{ \frac{\beta \phi_{t+1}}{\phi_t} c_t - \rho x_{1t} \right\} + \left\{ \frac{\beta \phi_{t+1}}{\phi_t} m_t - (1 - \rho)x_{2t}^m \right\} + \left\{ \frac{\beta \phi_{t+1}}{\phi_t} b_t + \beta(\psi_{t+1} + y)(1 - \theta_t)a_t - (1 - \rho)x_{2t}^b \right\} \geq 0
\]
and the currency, the reserves and the assets constraints,

\[ \frac{\beta \phi_{t+1}}{\phi_t} c_t - \rho x_{1t} \geq 0, \quad (3) \]

\[ \frac{\beta \phi_{t+1}}{\phi_t} m_t - (1 - \rho) x_{2t}^m \geq 0, \quad (4) \]

\[ \frac{\beta \phi_{t+1}}{\phi_t} b_t + \beta (\psi_{t+1} + y) (1 - \theta_t) a_t - (1 - \rho) x_{2t}^b \geq 0, \quad (5) \]

and the truth-telling constraints,

\[ x_{1t} \geq z_t x_{2t}^m + \kappa z_t x_{2t}^b, \quad (6) \]

\[ z_t (x_{2t}^m + x_{2t}^b) \geq x_{1t}, \quad (7) \]

and non-negative constraints,

\[ d_t, c_t, m_t, b_t, a_t, x_{1t}, x_{2t}^m, x_{2t}^b \geq 0. \quad (8) \]

The problem (1) subject to constraints (2)-(8) states that an insurance contract is chosen in equilibrium to maximize the expected utility of the representative buyer subject to the participation constraint for the insurance bank (2) and the resource constraints for currency, reserves and private assets (3)-(5) and the truth-telling incentive constraint for type 1 and 2 buyers, respectively, (6)-(7) and non-negativity constraints (8). In (1)-(8) \( d_t \) denotes deposit of buyers, \( c_t, m_t \) and \( b_t \) denote the quantities of currency, reserves and government bonds in terms of the CM good in the period \( t \) held by banks. \( a_t \) denotes the demand for the asset holdings of the bank and \( \psi_t \) denotes the price of the asset in the period \( t \) CM. \( x_{1t} \) denotes the consumptions of type 1 buyers in the period \( t \) DM, while \( x_{2t}^m \) and \( x_{2t}^b \) denote the consumptions of type 2 buyers in the period \( t \) DM based on reserves and the rest of assets, respectively, and \( x_{2t} \) is defined as the sum of \( x_{2t}^m \) and \( x_{2t}^b \).

The participation constraint (2) implies that the net payoff for the insurance bank must be non-negative. In the period \( t \) CM the insurance bank receive \( d_t \) deposits and invest in assets, and then provides \( x_{1t} \) amount of currency to type 1 buyers at the end of the CM and \( x_{2t} \) amount of consumption goods to agents who hold the deposit claims in the next period \( t + 1 \) CM. The currency, the reserves and the assets constraints (3)-(5) represent the resources of the insurance bank. Truth-telling incentive constraints (6)-(7) imply that each type of buyers prefers their own offers to the offers for the other types. Type 1 buyers can receive reserves and other assets by
mimicking type 2 buyers and try to transform these assets into currency. Similarly, type 2 buyers can receive currency by mimicking type 1 buyers and deposit it to the central bank as reserves. $1 - \kappa \in (0, 1)$ represents a proportional liquidation cost for selling government bonds and private assets to the central bank after the Walrasian market is closed.

From now on I focus on stationary equilibrium without time scripts on variables where $\frac{\phi_{t+1}}{\phi_t} = \frac{1}{\mu}$ holds for all time $t$ and $\mu$ denotes the gross inflation rate. From the maximization problem, the first-order conditions can be derived as

$$\frac{\mu}{\beta} = u'(x_1) + \frac{\lambda_1}{\rho} - \frac{\lambda_2}{\rho}, \quad (9)$$

$$z\frac{\mu}{\beta} = u'(x_2^m + x_2^b) - \frac{\lambda_1}{1 - \rho}z + \frac{\lambda_2}{1 - \rho}z, \quad (10)$$

$$q\frac{\mu}{\beta} = \frac{\psi}{\beta(\psi + y)} = u'(x_2^m + x_2^b) - \frac{\lambda_1}{1 - \rho}z + \frac{\lambda_2}{1 - \rho}z, \quad (11)$$

where $\lambda_1$ and $\lambda_2$ denote each multiplier associated with the constraints (6)-(7), respectively. In equilibrium asset markets clear in the CM with

$$a = A, \quad c = \phi C, \quad m = \phi M, \quad b = \phi B. \quad (12)$$

Since the supplies of currency and reserves are equal to the central bank’s government bonds and private asset holdings, we have

$$c + zm = (V - qb) + \psi \theta a, \quad (13)$$

and the proportion of currency and reserves among the total assets supply can be defined as $\delta$ by

$$c + zm = \delta(V + \psi a). \quad (14)$$

I assume that the central bank can set the level of the interest on reserves, $\frac{1}{z} - 1$, and choose the proportion of currency and reserves in the total asset supply, $\delta$, and the proportion of private assets held in the central bank, $\theta$, to implement monetary policy.

Finally, the quantity of government bonds held by the private sector must be less than or equal
to the total government bonds issued by the fiscal authority as

$$0 \leq qb \leq V.$$ (15)

**Definition 1.** Given parameters $$(\rho, y, V)$$ and the policy variables $$(z, \delta, \theta)$$, a stationary monetary equilibrium consists of quantities $$(x_1, x_2^m, x_2^b)$$ and prices $$(\mu, q, \psi)$$ and multipliers $$\lambda_1$$ and $$\lambda_2$$ which solve equations (9)-(15).

Since a quasi-linear utility is adopted in the model, the rates of return on assets such as currency, $$\frac{1}{\mu}$$; reserves, $$\frac{1}{z\mu}$$; government bonds, $$\frac{1}{q\mu}$$; and the Lucas tree, $$1 + \frac{y}{\psi}$$, cannot exceed the inverse of the time preference, $$\frac{1}{\beta}$$. In the model as long as the truth-telling incentive constraints do not bind, $$\lambda_1 = \lambda_2 = 0$$, reserves play the same role as the government bonds and the Lucas tree. So the rates of return on reserves must be equal or less than the rate of return on these assets in equilibrium. Moreover, we assume that the interest rate on reserves cannot be negative, i.e. $$z \leq 1$$, in equilibrium, because if the central bank sets a negative interest on reserves, then buyers will hold reserves as a form of currency for the higher rate of return. Thus, we have no arbitrage conditions in equilibrium as

$$\frac{1}{\mu} \leq \frac{1}{z\mu} \leq \frac{1}{q\mu} = \frac{\psi + y}{\psi} \leq \frac{1}{\beta}. \quad (16)$$

Since the utility and production functions are linear in the CM, all the surplus is generated from trades in the DM. So the welfare function is

$$W(x_1, x_2) = \rho\{u(x_1) - x_1\} + (1 - \rho)\{u(x_2) - x_2\} + yA, \quad (17)$$

which consists of the trading gains in the DM and the dividend from the Lucas tree. Note that the first-best equilibrium allocation is $$x_1 = x_2 = x^*$$, where $$x^*$$ satisfies with $$u'(x) = 1$$.

In the following analysis we assume that the total supply of assets in this economy is sufficiently small as $$V + \psi^f < x^*$$, where $$\psi^f$$ is defined as $$\psi^f = \frac{\beta y}{1 - \beta}$$, in order to make the first-best allocation infeasible.\(^\text{16}\) Note that there is no equilibrium case where only currency is scarce while the other assets are plentiful as shown in Champ et al. (1996), since the central bank is allowed to purchase private assets in this model.

\(^{16}\)If $$V + \psi^f \geq x^*$$ then we can always achieve $$x_1 = x_2 = x^*$$ by setting $$z = 1$$ and $$\delta \geq \frac{\beta y}{V + \psi^f}$$ in equilibrium, so there is no reason to consider monetary policy.
4 Monetary Equilibrium

In this section we characterize the equilibrium allocations only with the fixed consolidated government debt, $V$, by assuming $A = 0$, in order to focus on the effectiveness of monetary policy without considering the aggregate liquidity effect through the price of the private assets. Equilibrium cases can be distinguished by whether excess reserves exist or not, and also by which of the incentive constraints (6)-(7) bind or not.\(^{17}\)

4.1 Channel Systems

In our model we describe channel systems and floor systems as different equilibrium cases and will compare the effect of monetary policy and the welfare in these equilibrium allocations.

In a channel system the central bank sets the interest on reserves, $\frac{1}{2}z - 1$, lower than the nominal interest rate on government bonds, $\frac{1}{q} - 1$, which is controlled by open-market-operations, $\delta$. So banks will not hold any reserves in equilibrium, $m = 0$ and $x_m^2 = 0$, in this case. We also know that the constraints (3)-(5) always bind when the first-best allocation is not feasible. The first step is to solve for the equilibrium conditions without the binding truth-telling constraints (6)-(7). If $\lambda_1 = \lambda_2 = 0$, then the first-order conditions (9) and (11) can be reduced into

\[ \frac{\mu}{\beta} = u'(x_1), \quad (18) \]
\[ q \frac{\mu}{\beta} = u'(x_2), \quad (19) \]

respectively. By using these first-order conditions (18)-(19) and the binding constraints (3) and (5), the government budget constraint (13) with $A = 0$ can be transformed into a feasibility condition,

\[ \rho x_1 u'(x_1) + (1 - \rho)x_2 u'(x_2) = V. \quad (20) \]

Note that given a nominal interest rate target $\frac{1}{q} - 1$, $x_1$ and $x_2$ have a strictly positive relationship as $qu'(x_1) = u'(x_2)$ in (18)-(19) and a strictly negative relationship in (20). So there is a unique equilibrium allocation $(x_1, x_2)$ which satisfies with (18)-(20): The point $E$ in Figure 2 describes the equilibrium allocation $(x_1, x_2)$ that intersects $FOC$ curve from (18)-(19) and $IC$ curve (20).

In order to support this equilibrium allocation $(x_1, x_2, q)$, the central bank can implement open-

\(^{17}\)In the following paper channel and floor systems are the different types of equilibrium, which can be distinguished by the amount of excess reserves in the banking sector.
market-operations by choosing $\delta$ as

$$\delta = \frac{\rho x_1 u'(x_1)}{V},$$

(21)

which can be derived from (14) and (20).

In Figure 2, given $z \leq 1$ the equilibrium allocations $(x_1, x_2)$ are feasible on the IC curve between the points B and D associated with $q \in [\tilde{q}, z]$ where $\tilde{q}$ is defined as the lower bound of $q$ at the point D. Since $\delta$ is increasing in $x_1$ in (21), each $q \in [\tilde{q}, z]$ is implemented by a corresponding $\delta \in [\tilde{\delta}, \bar{\delta}]$, where $\tilde{\delta}$ and $\bar{\delta}$ are the lower and upper bound of $\delta$: Given a specific utility function, $u(x) = x_1^{1-\gamma} - x_2$, $q = \left(\frac{1-\rho}{\rho} \frac{\delta}{1-\gamma}\right)^{\frac{1}{1-\gamma}}$ can derived from (18)-(21). As shown in Figure 2, when $(\tilde{x}_1, \tilde{x}_2)$ and $(\bar{x}_1, \bar{x}_2)$ are defined as the allocations at $q = \tilde{q}$ and $q = z = 1$, respectively, each $\tilde{\delta}$ and $\bar{\delta}$ satisfies (21) with $\tilde{x}_1$ and $\bar{x}_1$, respectively.\(^{18}\)

Lemma 1. In channel systems the truth-telling constraints (6)-(7) do not bind.

Proof. Given a specific utility function, $u(x) = x_1^{1-\gamma}$ where $\gamma < 1$, the first-order conditions (18)-(19) can be rewritten as $x_2 = \frac{1}{q^\gamma} x_1$. Since $q < z \leq 1$ in channel systems, $x_2 = \frac{1}{q} x_1 > \frac{1}{2} x_1$ always holds, so the truth-telling constraint (7) does not bind in equilibrium. Similarly, if $\kappa$ is sufficiently small as $\kappa \leq \tilde{q}^{\frac{1}{\gamma} - 1}$ holds, the truth-telling constraint for type 1 buyers (6) does not bind in channel systems because $x_1 = q^\gamma x_2 > \kappa x_2$ holds in equilibrium. QED

\(^{18}\)Note that in case of the zero lower bound, $q = 1$, $\delta > \tilde{\delta}$ can also support the same equilibrium allocation with $\delta = \bar{\delta}$, because the reserves can be held at $q = z = 1$. 

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Lemma 1 shows that the truth-telling incentive constraints do not bind in the channel system. The insurance contract provides currency for type 1 buyers and government bonds for type 2 buyers to make the marginal rate of substitution between currency and government bonds equivalent with the given nominal interest rate. Without the excess reserves, type 1 buyers prefer currency because it is too costly to transform the government bonds into currency. Type 2 buyers prefer government bonds as long as the value of government bonds is greater than the value of currency plus the expected interest payments. So, the optimal insurance contract does not violate the truth-telling constraints.

In a channel system monetary policy is implemented only by open-market operations, because the level of the interest rate on reserves is irrelevant to determine the equilibrium allocations. For example, suppose that the central bank tries to raise the nominal interest rate, \( q \), by reducing \( \delta \), i.e. open-market sales. In Figure 2 IC curve remains, but FOC curve shifts to the left. Since currency is absorbed and the government bonds are injected, the consumption in the currency trade, \( x_1 \), falls while the consumption in the collateral transaction, \( x_2 \), rises. The liquidity premium of currency goes up whereas the liquidity premium of the government bonds falls, thus both the inflation rate and the real interest rate on government bonds increase, and then the nominal interest rate also rises by the Fisher equation.

Finally, open-market operations are ineffective at the zero lower bound, \( q = 1 \). With zero interest on reserves, \( q = z = 1 \) holds at the zero lower bound.\(^{19}\) Since the rates of return on reserves and government bonds are equal at \( q = z = 1 \), we can have \( m \geq 0 \) and \( x_2^m \geq 0 \) in this case. The allocation \((x_1, x_2)\) is determined from (18)-(19) with \( q = 1 \), but reserves and government bonds are indeterminate in equilibrium from (13)-(14) and (20)-(21), \( m + b = V - \rho x_1 u'(x_1) \). Thus, if \( \delta \geq \bar{\delta} \), there is a liquidity trap where adjusting \( \delta \) is no longer effective.

### 4.2 Floor Systems

In a floor system the interest on reserves must be greater than or equal to the nominal interest rate, \( z \leq q \), to have excess reserves. Given the interest on reserves, \( \frac{1}{z} - 1 \), if \( \delta = \bar{\delta} \) then we have the same equilibrium as in channel systems with \( m = 0 \) and \( x_2^m = 0 \), so when \( \delta > \bar{\delta} \) banks hold excess reserves on their balance sheets as \( m > 0 \) and \( x_2^m > 0 \). Therefore, in order to implement floor systems, the central bank can set \( z \leq 1 \) and \( \delta > \bar{\delta} \) where \( \bar{\delta} \) satisfies with \( z = \left( \frac{1 - \rho \delta}{\rho} \right)^{1-\gamma} \),

given the utility function \( u(x) = x^{1-\gamma} \).

\(^{19}\)In this respect the equilibrium at the zero lower bound has a similar feature to the floor system.
Define $\alpha$ as a proportion of type 2 buyer’s consumption supported by excess reserves, $\alpha = \frac{x_2 m}{x_2 m + x_2 b}$. Note that $\alpha \in [0, 1]$ increases in $\delta$ and corresponds to $\delta \in [\bar{\delta}, 1]$ as $\alpha = \frac{\delta - \bar{\delta}}{1 - \bar{\delta}}$ from the binding constraints (3)-(5) and (13)-(14). Then the truth-telling constraint (6) can be rewritten as

$$x_1 \geq \alpha \kappa z x_2,$$

where $\alpha \kappa$ is defined as $\alpha \kappa = \alpha + \kappa (1 - \alpha)$.

**Lemma 2.** In the floor system the truth-telling constraint (7) never binds, but the truth-telling constraint (6) can bind when the excess reserves are sufficiently large.\(^{21}\)

**Proof.** When $\lambda_1 = \lambda_2 = 0$, the equilibrium conditions (18)-(20) still hold given $q = z < 1$, so we have $x_1 = z^{\frac{1}{\gamma}} x_2$ in equilibrium. The truth-telling constraint for type 2 buyers (7) does not bind because $\frac{1}{q} > \frac{1}{z}$ at $q = z < 1$. However, the truth-telling constraint for type 1 buyers (6) can bind when the excess reserves are sufficiently large. Given $z < 1$ there exists a threshold $\hat{\alpha} \in (0, 1)$ which satisfies with $\hat{\alpha} + \kappa (1 - \hat{\alpha}) = z^{\frac{1}{\gamma}} - 1$ from (22). Since $\hat{\alpha}$ is associated with $\hat{\delta}$ as $\frac{\hat{\delta} - \bar{\delta}}{1 - \bar{\delta}}$, we know that (22) binds with $\delta \in (\hat{\delta}, 1)$, which is equivalent with $\alpha \in (\hat{\alpha}, 1]$. Note that $\hat{\delta} > \bar{\delta}$ because (22) does not bind at $\alpha = 0$ because $\kappa$ is sufficiently small as $\kappa \leq \tilde{q}^{\frac{1}{\gamma}} - 1 \leq z^{\frac{1}{\gamma}} - 1$. QED

Lemma 2 shows that the truth-telling constraint for type 1 buyers can bind when the excess reserves are sufficiently large. In a floor system the interest on reserves plays a role as the nominal interest rate to adjust the marginal rate of substitution between currency and government bonds. However, it is allowed that currency and reserves can be transformed each other without an additional cost. So, the optimal allocation cannot be obtained when the large excess reserves are provided to type 2 buyers, because type 1 buyers can mimic type 2 buyers.

Note that type 1 buyers deviates in this model unlike Diamond and Dybvig (1983) where type 2 buyers deviates, because we assume that the coefficient of relative risk aversion, $\gamma$, is less than 1. Since the insurance contract makes type 1 buyers worse off and type 2 buyers better off with $\gamma < 1$, type 1 buyers tends to deviate and truth-telling constraint (6) can bind here.

Therefore, we can describe our equilibrium cases as shown in Figure 3. Given $z < 1$, if $\tilde{\delta} \leq \delta \leq \bar{\delta}$ then we can conduct open-market operations in channel systems. Given $z < 1$, if $\delta > \bar{\delta}$ we need to implement monetary policy in floor systems. In case of $\delta < \bar{\delta} \leq \tilde{\delta}$ then the truth-telling constraint (22) does not bind, but it binds in case of $\tilde{\delta} < \delta \leq 1$. Note that given $(z, \bar{\delta}), \bar{\delta}$ can be derived from

---

\(^{20}\)Note that $c = \rho x_1 u'(x_1) = \tilde{\delta} V$, $zm = \alpha (1 - \rho) x_2 u'(x_2) = (\delta - \bar{\delta}) V$, and $qb = (1 - \alpha) (1 - \rho) x_1 u'(x_1) = (1 - \delta) V$ hold from (3)-(5) and (13)-(14).

\(^{21}\)As long as $\kappa < 1$, both constraints (6)-(7) cannot bind simultaneously with $x_2^m, x_2^b \geq 0$. 

---
\[ \hat{\alpha} = \frac{\hat{\delta} - \bar{\delta}}{1 - \bar{\delta}} \text{ and } \hat{\alpha} + \kappa (1 - \hat{\alpha}) = z^{\frac{1}{\gamma} - 1}. \]

4.3 **Floor System with Liquidity Trap**

When \( \lambda_1 = 0 \), the equilibrium allocation at \( z < 1 \) in a floor system is the same as one in a channel system with \( q = z < 1 \), because given \( \lambda_1 = 0 \), \( q = z \) holds in (10)-(11) and the equilibrium conditions (18)-(20) still hold. In this case reserves are used as a perfect substitute of government bonds and the rates of return on both assets are equal. As shown in the zero-lower-bound case in the channel system, since reserves and government bonds are indeterminate in equilibrium as \( m + b = \frac{V - \rho x_1 u'(x_1)}{2} \) from (13)-(14) and (20)-(21), open-market operations are no longer effective in this case.

However, the nominal interest rate target, \( \frac{1}{q} - 1 \), can be always achieved by adjusting the level of the interest rate on reserves, \( \frac{1}{z} - 1 \). So, the interest on reserves is effective and we can obtain the same allocation by raising \( \frac{1}{z} - 1 \) in the floor system instead of reducing \( \delta \) in the channel system. Therefore, the impacts of monetary policy in both systems on the real interest rate and/or the inflation rate are equivalent. Note that although the equilibrium allocations are identical, the implementation mechanism in the floor system is different from the one in the channel system. In the channel system the real quantities of currency and government bonds pin down the rates of return on currency and government bonds, whereas in the floor system the relative price between

\[ 22 \delta_{z=q} = (1 - \hat{\delta}) \delta^{\frac{1}{\gamma} - 1 - \kappa} + \tilde{\delta} \text{ could be either larger or smaller than } \rho. \]
currency and reserves, i.e. the interest rate on reserves, determines the demands for currency and reserves, respectively.

4.4 Floor System without Liquidity Trap

In a floor system when \( \lambda_1 > 0 \), the feasibility condition (20) does not change, but the truth-telling constraint (22) binds with equality instead of the first-order conditions (18)-(19).\(^{23}\) Thus, in this case given \((z, \delta)\), the equilibrium allocation \((x_1, x_2)\) in the floor system with \( \lambda_1 > 0 \) is determined by (20) and (22) with equality. Define \((x_1^i, x_2^i)\) as the equilibrium allocations in the channel system\((c)\) and floor system\((f)\), respectively, where \(i = \{c, f\}\). As shown in Figure 4, given the same level of interest on reserves, \(\frac{1}{z} - 1\), the equilibrium allocation \((x_1^f, x_2^f)\) in the floor system with \( \lambda_1 > 0 \) can be described as the point \(F\) while the equilibrium allocation \((x_1^c, x_2^c)\) in the channel system(or in the floor system with \( \lambda_1 = 0 \)) can be shown as the point \(C\). Notice that \(x_1^c < x_1^f\) and \(x_2^c > x_2^f\) in the graph, because \((x_1^c, x_2^c)\) and \((x_1^f, x_2^f)\) still satisfy with the same feasibility condition (20) while (22) must violates at \((x_1^c, x_2^c)\) when \( \lambda_1 > 0 \).

![Figure 4. Equilibrium in Floor System without Liquidity Trap](image)

Therefore, given the interest on reserves, \(\frac{1}{z} - 1\), the inflation rate and the real interest rate on government bonds can be changed from the ones in the channel system as \( \delta \) increases. When \(^{23}\)Note that (20) does not change because \( \lambda_1 \) terms are cancelled out when the binding constraints (3)-(5) and the first-order conditions (9)-(11) are plugged into the government budget constraint (13).
\( \lambda_1 > 0 \), from (9)-(11) we obtain

\[
\frac{1}{\beta r_m^f} := z\frac{\mu_f^f}{\beta} = \rho z u'(x_1^f) + (1 - \rho) u'(x_2^f),
\]

(23)

\[
\frac{1}{\beta r_b^f} := q\frac{\mu_f^f}{\beta} = \rho\kappa z u'(x_1^f) + (1 - \rho\kappa) u'(x_2^f).
\]

(24)

where \( \mu^i, r_m^i, r_b^i \) denote the inflation rate, the rate of return on reserves, and the rate of return on government bonds in the channel system (c) and floor system (f), respectively, where \( i = \{c, f\} \).

In this case the liquidity premia of currency and government bonds do not depend only on the corresponding type market condition: The liquidity premium in each asset is calculated as a weighted average of the marginal utility in both currency and collateral trades as shown in (23)-(24). Therefore, the liquidity premia of currency and government bonds can move into the same direction.

Given \( \lambda_1 > 0 \), since the point \( C \) is not achieved, the marginal rate of substitution between type 1 buyer’s consumption and type 2 buyer’s consumption, \( \frac{u'(x_1^f)}{u'(x_2^f)} \), is lower than the interest on reserves, \( \frac{1}{z} \). It is optimal for the insurance banks to raise \( x_2 \) and reduce \( x_1 \), but it is not feasible because they cannot reveal the types under private information. Therefore, there arises a positive liquidity premium on currency in (9) while a negative liquidity premium on reserves and government bonds in (10)-(11) with \( \lambda_1 > 0 \). Moreover, the liquidity premium on government bonds is greater than the liquidity premium on reserves with \( \kappa < 1 \), because government bonds are more useful than reserves to reveal the types. Therefore, in equilibrium we have a return dominance between reserves and government bonds, \( q > z \), as shown in (23)-(24) because \( \kappa < 1 \) and \( u'(x_2) > z u'(x_1) \) holds when (22) binds. Thus, open-market operations, i.e. the exchange between reserves and government bonds, can be effective in this case.

### 4.4.1 Interest on Reserves (\( \Delta \frac{1}{z} \))

Notice that we have two policy variables, the proportion of currency and reserves, \( \delta \) (or \( \alpha \)), and the interest on reserves, \( \frac{1}{z} - 1 \). Given \( \delta \), if the interest rate on reserves, \( \frac{1}{z} - 1 \), is raised, then the equilibrium allocation \((x_1^f, x_2^f)\) moves from the point \( F \) toward the point \( C \) in Figure 4 as FOC curve (22) shifts to the left. On the other hand, given \( z \) if the proportion of currency and reserves, \( \delta \), is raised by open-market operations, the equilibrium allocation \((x_1^f, x_2^f)\) also moves toward the
point $C$ in Figure 4. However, their impacts on the inflation rate and the real interest rate are quite different.

**Proposition 1.** Given $\delta$, both the inflation rate and the real interest rate on government bonds increase in the interest of reserves, $\frac{1}{z} - 1$, in the floor system without liquidity trap.

**Proof.** By plugging (20) and (22) into (23)-(24), we can have

$$\frac{\mu}{\beta} = \rho u'(x_1) + \frac{1-\rho}{z} u'(x_2) = (1-\alpha\kappa)\rho u'(x_1) + \frac{\alpha\kappa V}{x_1},$$

$$\frac{1}{\beta r^b} = z\rho\kappa u'(x_1) + (1-\rho\kappa)u'(x_2) = \alpha\kappa V x_2 + \frac{\alpha\kappa u'(x_1)}{x_2}.$$ (25)

Given $\delta$, if $z$ is reduced then $x_1$ decreases and $x_2$ increases along the IC curve (20), so both $\mu$ and $r^b$ increase in (25). QED

When the interest on reserves goes up, the truth-telling incentive constraint (22) is tightened with higher $\lambda_1$. Therefore, the liquidity premium in currency goes up while the liquidity premium in government bonds decreases. So, both the inflation rate and the real interest rate on government bonds increase.

### 4.4.2 Open-market Operations($\Delta \delta$)

**Proposition 2.** Given $z$, the inflation rate decreases in $\delta$ while the real interest rate on government bonds increases in $\delta$ in the floor system without liquidity trap.

**Proof.** Given $z$, if $\delta$ decreases, then the equilibrium allocation $(x_1, x_2)$ moves toward the point $C$ in Figure 4. We can check the changes in the inflation rate and the real interest rate by reducing $x_1$ along the curve (20) given the same $z$.

$$\frac{\partial \mu}{\partial x_1} = \beta \{\rho u''(x_1) + (1-\rho)\frac{u''(x_2)}{x_2} \frac{\partial x_2}{\partial x_1} | V \} = \beta \rho \gamma u'(x_1) \{-\frac{1}{x_1} + \frac{1}{x_2} \} < 0,$$

$$\frac{\partial q\mu}{\partial x_1} = \beta \{\rho\kappa u''(x_1) + (1-\rho\kappa)u''(x_2) \frac{\partial x_2}{\partial x_1} | V \} = \beta \rho \kappa \gamma u'(x_1) \{-\frac{z}{x_1} + \frac{1}{x_2} \} < 0.$$ (26)

In (26) we use $\frac{\partial x_2}{\partial x_1} | V = -\frac{\rho u'(x_1)}{(1-\rho)u'(x_2)}$ from (20), given $\frac{-xu''(x)}{u(x)} = \gamma$. Thus, the inflation rate goes up while the real interest rate on government bonds decreases when $x_1$ decreases by reducing $\delta$. QED

In this case there can be two different effects on the liquidity premia in currency and government bonds. Open-market purchases increase the quantity of reserves while decrease the quantity of government bonds in the market. Since the truth-telling incentive constraint (22) is tightened further, the insurance bank provides more currency to type 1 buyers while less reserves and government bonds to type 2 buyers. By the liquidity supply effect, the marginal utility in currency trade
goes down while the marginal utility in collateral trade increases as described in channel system. However, unlike the channel system, the liquidity premia on both assets are derived as the weighted average of the marginal utilities in the trading markets as shown in (23)-(24). Consequently, both the liquidity premia on currency and government bonds fall as the gap between type 1 and 2 consumption becomes smaller.\footnote{The inflation rate and the real interest rate move in different directions like the Mundell-Tobin effect, in which the inflation rate increases by raising money supply while the real interest rate decreases because the demand for currency diminishes, but both logic and result in this paper are different from it.}

The effects of monetary policy implementations on the inflation rate and the real interest rates by different cases can be summarized in Table 1.

<table>
<thead>
<tr>
<th>Regime (Policy)</th>
<th>Channel ((z, \delta))</th>
<th>Floor(LT) ((z, \delta))</th>
<th>Floor(w/o LT) ((z, \delta))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target</td>
<td>(\mu) ((x, -))</td>
<td>((-x, x))</td>
<td>((-x, -))</td>
</tr>
<tr>
<td></td>
<td>(r_m) ((x, -))</td>
<td>((-x, x))</td>
<td>((-x, +))</td>
</tr>
<tr>
<td></td>
<td>(r_b) ((x, -))</td>
<td>((-x, x))</td>
<td>((-x, +))</td>
</tr>
</tbody>
</table>
respectively. From (28) we can have

$$\frac{\partial \mu^f}{\partial z} = \frac{\partial \mu^c}{\partial z} \left( \rho \alpha_\kappa \gamma z^{\gamma-1} + (1-\rho) \right)^{1/\gamma} \left[ \gamma + (1-\gamma) \frac{\rho \alpha_\kappa + (1-\rho) \alpha_\kappa \gamma z^{\gamma-1}}{\rho + (1-\rho) \alpha_\kappa z^{\gamma-1}} \right].$$  (29)

Given the same level of $z$, since $x^f_1 < x^f_2$, $z^{1-\frac{1}{\gamma}} > \alpha_\kappa^{\gamma-1} z^{\gamma-1}$ holds in (27).26 By using this inequality, we can find out that $|\frac{\partial \mu^f}{\partial z}| < |\frac{\partial \mu^c}{\partial z}|$ holds in (29). Since the Fisher equation holds in the model, the effect on the real interest rate in the floor system without liquidity trap is greater than one in the floor system with liquidity trap. QED

Proposition 3 shows that raising the interest on reserves in the floor system without liquidity trap can increase the real interest rate more than that in the floor system with liquidity trap. If the truth-telling incentive constraint binds, the constraint is tightened with higher $\lambda_1$ when the interest on reserves goes up. Thus, it raises the inflation further in (9) and reduces the real interest rate on government bonds further in (11).

### 4.4.4 Liquidation Cost ($\triangle \kappa$)

Except for the interest on reserves and open-market operations, if the central bank can adjust the liquidation cost, $1 - \kappa$, in the discount window facility, then it could be used as another policy variable in the floor system without liquidity trap.27 When the liquidation cost, $1 - \kappa$, decreases, the truth-telling constraint is tightened, so the equilibrium allocation moves from the point $C$ to the point $F$ in the Figure 4. Therefore, the effects on the inflation and the real interest rate are similar to the case of raising $\delta$. However, the gap between the rates of return on reserves and government bonds will be reduced in (23)-(24).

**Corollary 1.** At the same level of $\delta$ and $z$, when the liquidation cost, $1 - \kappa$, decreases, both the inflation rate and the real interest rate on government bonds decrease in the floor system with liquidity trap.

**Proof.** Given $\delta$ and $z$, if $\kappa$ increases, then the equilibrium allocation $(x^f_1, x^f_2)$ moves toward the point $F$ in Figure 4. As shown in (26) at the proof for Proposition 2, the inflation rate goes down as $x^f_1$ increases. In case of the the real interest rate on government bonds, we need to consider not

26 This inequality, $z^{1/\gamma} < \alpha_\kappa z$, implies that the slope of the first-order condition (18)-(19) in the channel system(or the floor system with liquidity trap) is greater than the slope of the first-order condition (22) in the floor system without liquidity trap in Figure 4.

27 Note that the liquidation cost is not associated with the equilibrium conditions in the channel system and the floor system with liquidity trap as shown in the previous sections.

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only the movement of the allocation \((x_t^1, x_t^2)\), but also the change in the weight with \(\kappa\) in (24).

\[
\frac{\partial q_{\mu}}{\partial \kappa} = \beta\rho\{zu'(x_t^1) - u'(x_t^2)\} + \frac{\partial q_{\mu}}{\partial x_t^1}|_{\kappa} \frac{\partial x_t^f}{\partial \kappa} < 0.
\]

We know that given \(\kappa\), \(\frac{\partial q_{\mu}}{\partial x_t^1} < 0\) from (26) and \(zu'(x_t^1) < u'(x_t^2)\) holds at the point \(F\). Therefore, the real interest rate on government bonds increases in the liquidation cost \(\kappa\). QED

Since the real value of the consolidated government debt is fixed as shown in (20), there is no aggregate supply effect of monetary policy in the model. Therefore, if there exists a welfare-maximizing point on the curve (20), we can achieve the optimal equilibrium allocation in any regime by choosing the different levels of the interest on reserves and/or the quantity of reserves.

5 Equilibrium with Private Assets

In real world banks invest not only in nominal government debt such as currency, reserves, and government bonds, but also in real assets. In this section we extend the model with private assets, \(A > 0\), in order to study the aggregate supply effect, which is associated with the asset prices. Given \(A > 0\), instead of (20), we can obtain

\[
\rho x_1 u'(x_1) + (1 - \rho)x_2 u'(x_2) = V + \psi A.
\]

from the binding constraints (3) and (5), government budget constraint (13) and the first-order conditions (18)-(19) in the channel system, where \(\psi = \frac{\beta yu'(x_2)}{1 - \beta w'(x_2)}\) holds from (11). In the floor system, even with \(\lambda_1 > 0\), we still have (30) from (3)-(5), (13) and the first-order condition (22), but in this case \(\psi = \frac{\beta yu'(x_1) + (1 - \rho)u'(x_2)}{1 - \beta (\rho ku'(x_1) + (1 - \rho)w'(x_2))}\) holds from (11). Therefore, the equilibrium allocation \((x_1, x_2)\) is uniquely determined from (18)-(19) and (30) with \(\psi = \frac{\beta yu'(x_2)}{1 - \beta w'(x_2)}\) in the channel system and the floor system with liquidity trap, while determined from (22) and (30) with \(\psi = \frac{\beta yu'(x_1) + (1 - \rho)u'(x_2)}{1 - \beta (\rho ku'(x_1) + (1 - \rho)w'(x_2))}\) in the floor system without liquidity trap.

Notice that the proportion of private assets held by the central bank, \(\theta \in [0,1]\), is irrelevant to determine the equilibrium allocation as long as (15) does not violate, because both government bonds and private assets are traded as identical securities by type 2 agents in the model. In order to maintain this feature, I assume that when \(\delta (V + \psi A) > V\) holds from (14)-(15), the central bank

\[28\]The only difference is that the aggregate supply of private assets can be changed by the asset prices, while the real quantity of government bonds is fixed.
will choose $\theta \in [\bar{\theta}, 1]$ where $\bar{\theta} = \delta (1 + \frac{V}{\delta \nu_{y}(x_{2})}) - 1$ is derived from (13)-(14) with $qb = 0$.

Unlike the previous cases of $A = 0$, the feasible equilibrium allocation set can be expanded or restricted by the level of the asset price, $\psi$, in (30). For example, if the real interest rates in the channel system, $r_{c}^{b} = \delta$, in (19) and the real interest rates in the floor system without liquidity trap, $r_{b}^{f}$, in (24) increase, then the asset price falls in (11), so the feasibility condition (30) shrinks toward the origin. If the central bank implement monetary policy in the channel system with open-market operations, $\delta \in [\tilde{\delta}, \rho]$, at zero interest on reserves, $z = 1$, then the feasibility condition (30) can be described as $IC(V + \psi)$ curve in Figure 5a. Note that the slope of $IC(V + \psi)$ curve (30) is less steep than the slope of $IC(V)$ curve (20), because when $\delta$ increases, the real interest rate goes down.

It is not simple in case of floor system without liquidity trap, because the central bank can use either the interest on reserves($\frac{1}{z} - 1$) or open market operation($\tilde{\delta}$) and also the effect on the real interest rate can vary by these policy tools. Therefore, we consider the shape of the feasibility curve (30) for each policy variables, $z$ and $\delta$, by considering the effects of the policy variables on the real interest rates, respectively.

5.1 Open-market Operations($\Delta \delta$)

In Figure 3 given $z \in (\bar{q}, 1)$, as the central bank raises $\delta$ from $\tilde{\delta}$ to 1, the equilibrium allocation starts with the channel system regime and passes the floor system with liquidity trap regime and reaches floor system without liquidity trap regime. Since the open-market operations are ineffective in the floor with liquidity trap regime, in Figure 5a once the equilibrium allocation arrives at the point $H$ with $\delta = \hat{\delta}$, it remains when $\delta$ increases further. When $\delta$ reaches $\hat{\delta}$ in the floor system without liquidity trap regime, raising $\delta$ tightens the binding incentive constraint (22), so the real interest rate goes up and the asset price falls. Therefore, we have a kink at the point $H$ and the feasibility of equilibrium allocations in the floor system without liquidity trap becomes restricted more than that in the channel system.

Corollary 2. When $\delta$ is raised, the feasible set of equilibrium allocation is reduced further in the floor system without liquidity trap than in the channel system.

Proof. By Proposition 2, the real interest rate on private assets goes up when $\delta$ increases in the floor system without liquidity trap. Therefore, the asset price decreases and the feasibility condition moves toward to the origin. QED
As summarized in Table 1, when $\delta$ increases, both the inflation rate and the real interest rate decrease in the channel system whereas the inflation rate falls and the real interest rate goes up in the floor system without liquidity trap, so we can describe the inflation rate and the rate of return on government bonds as shown in Figure 5b.

5.2 Interest on Reserves($\Delta \frac{1}{z}$)

Similarly, given $\delta \in (\rho, 1)$ when the central bank reduces $z$ from 1 to $\tilde{q}$, the equilibrium allocation starts with the floor system with liquidity trap and reaches the floor system without liquidity trap as shown in Figure 3. Since changing the interest on reserves in the floor system with liquidity trap has the same effect as in the channel system, when $z$ is reduced, the equilibrium allocation moves along the curve (30) from the point $I$ to the point $J$ in Figure 6a. However, when $\frac{1}{z} - 1$ increases, the real interest rate in the floor system without liquidity trap goes up further than one in the floor system with liquidity trap as shown in Proposition 3. Thus, the asset prices go down further and the feasible allocation set shrinks as described in Figure 6a.

**Corollary 3.** When the interest on reserves, $\frac{1}{z} - 1$, is raised, the feasible set of equilibrium allocation is reduced further in the floor system without liquidity trap than in the floor system with liquidity trap.
Proof. By Proposition 3, when the interest on reserves, $\frac{1}{z} - 1$, increases, the real interest rate on private assets goes up further in the floor system without liquidity trap comparing to the floor system with liquidity trap. Therefore, the asset price decreases further and the feasible set of the equilibrium allocation is more restricted. QED

When $\frac{1}{z} - 1$ is raised, both the inflation rate and the real interest rate increase either in the floor system with liquidity trap or in the floor system without liquidity trap. However, both the inflation rate and the real interest rate rise further in floor system without liquidity trap as described in Figure 6b.

As shown in Figures 5a and 6a, raising $\delta$ and reducing $z$ in the floor system without liquidity trap regime shrink the set of the feasible equilibrium allocations. So, if given $(\delta, z)$ an equilibrium allocation is located at the floor system without liquidity trap in Figure 3, there is a possibility to improve welfare by expanding the feasibility set. For example, by reducing $\delta$ to $\bar{\delta}$, and then raising $z$ in the floor system with liquidity trap regime, the allocation can move from the point $G$ to the point $H$, and then to the point $G'$ in Figure 5a. Similarly, by raising $z$ to reach the floor system with liquidity trap regime, and then reducing $\delta$ to the channel system in Figure 3, the allocation can move from the point $K$ to the point $J$, and then to the point $K'$ in Figure 6a. If there are no restrictions on the policy variables $(\delta, z)$, the channel system with $\delta \in (\hat{\delta}, \rho]$ and $z = 1$ is desirable to maximize the feasibility of the equilibrium allocations.
5.3 Discussion

We find out that when the truth-telling constraint binds with sufficiently large excess reserves, open-market operations can be effective in the real economy. However, this result is not desirable in the welfare perspective, because both policy variables reduce the feasibility of the equilibrium allocations. Therefore, if the large amount of excess reserves is inevitable, there could be an alternative solution: Instead of liquid excess reserves, the central bank can issue illiquid bonds such as the central bank’s bill or ON-RRP. Issuing the illiquid debt can prevent the banks holding too much liquidity in their portfolio. Moreover, raising the liquidation cost, $1 - \kappa$, of government bonds is shown as useful for separating the types in the model. In this respect, maintaining the width of the corridor wide could be helpful when we implement the floor system with excess reserves.

6 Conclusion

In this paper we study the effectiveness of monetary policy tools in the floor system, when core banks provide a liquidity insurance to periphery banks. Since excess reserves can inhibit the core banks separating the types by liquidity needs, there could exist a new regime in the floor system where open-market operations can be effective with a return dominance. When we consider the aggregate liquidity effect with private assets, the set of feasible equilibrium allocations can be reduced in this regime. In this case we can expand the feasibility by reducing either the interest on reserves or the proportion of excess reserves in the bank’s balance sheet or both.

This paper takes a step forward to understand monetary policy with excess reserves. It provides a theoretical model in which we can implement both the interest on reserves and open-market operations in the floor system. However, it also leaves some questions unanswered. For example, this model uses an insurance contract with truth-telling constraints instead of a secondary financial markets under private information. In this respect we may ask how the secondary market friction can also influence the effectiveness of these monetary policy tools. Moreover, since the truth-telling constraints can prevent bank runs, we may study further how the fragility of banks is associated with the effectiveness of monetary policy. Finally, there are other respects of excess reserves which could be considered in the future work. For example, excess reserves can be helpful for banks when they are exposed to an aggregate liquidity shock.

\footnote{Overnight reverse repurchase agreement (ON-RRP) facility sells a security to an eligible counterparty and simultaneously agrees to buy the security back the next day.}
References


