Scarcity of Assets, Private Information, and the Liquidity Trap*

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Abstract

This paper explores how private information can restrict liquidity insurance and the implementation of monetary policy. A general equilibrium banking model is constructed in which banks provide liquidity insurance under lack of memory. Given idiosyncratic liquidity risks it is beneficial to separate the types by liquidity needs; however, it is difficult when the type information is private. A banking arrangement is considered in which two different liquid assets are used for revealing the types. When a truth-telling constraint binds with scarcity of the illiquid assets, a proportion of liquid assets can be held in balance sheets of banks to reveal the types. Liquidity trap equilibrium, in which exchanging liquid assets with illiquid assets is no longer effective, can exist away from the zero-lower-bound.

Key Words: truth-telling constraints, separating equilibrium, liquidity insurance JEL Codes: E4, E5.

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1 Introduction

The liquidity trap, in which monetary policy is no longer effective, has been a subject of interest to both monetary theorists and central bankers since the Great Depression of the 1930s. In particular, excess reserves are associated with the liquidity trap because exchanging reserves with government bonds is ineffective to real allocations. One conventional explanation for excess reserves is based on lack of good loan opportunities. For example, in recessions commercial banks would hold excess reserves voluntarily since the expected return of projects is lower than the rate of return in reserves. The other widespread view of excess reserves is that banks accumulate reserves to offset the aggregate liquidity shock such as large withdrawals in banking panics. In this paper I develop a new theory of the liquidity trap in which excess reserves are useful for providing liquidity insurance efficiently by revealing private information.

Providing liquidity insurance is one of the primary functions of banks. When people are exposed to idiosyncratic liquidity risk, separating the types by liquidity needs ex post is beneficial for efficient liquidity distribution. For example, a banking contract can provide liquid assets to impatient agents with low returns while promising high returns to patient agents with illiquid assets. If the returns for the patient types are sufficiently high, private information on the types does not matter because the patient types would prefer to earn high returns. However, if the rate of return for patient types is sufficiently low then the patient types have an incentive to mimic impatient types so that separating the types is difficult under private information. In this case in order to separate the types, banks are required to hold sufficient assets, either liquid or illiquid, for patient types. In this respect excess reserves can exist in the balance sheet of banks to distribute liquidity efficiently by types. In this case if the real value of total assets is fixed, adjusting the proportion of liquid and illiquid assets is no longer effective in real allocations.

In order to explore this issue I construct an asset-exchange model in which two different liquid assets can be used for separating the types. This micro-founded model is useful for incorporating informational frictions such as lack of memory and limited commitment, and it is also highly tractable with a banking contract. The basic structure of the model builds on Lagos and Wright (2005), specifically Rocheteau and Wright (2005), where ex ante heterogeneous agents trade in decentralized meetings and rebalance their portfolios in a centralized market. The form of banking contract comes from Williamson (2012) where a banking contract provides liquid assets for asset exchange and illiquid assets for credit arrangements. There are fixed supplys of both private and government assets that are insufficient to support the optimal level of consumption in both exchanges. Given the supply of assets, a banking contract is considered to maximize the ex ante expected value of depositors under perfect competition.

One important assumption is lack of record-keeping technology. This anonymity assumption makes recognizable assets essential for decentralized trade. Simultaneously, when agents are subject
to individual liquidity risk, this imperfect memory inhibits banks in revealing the types.\footnote{If record-keeping is available, credit or (proportional) tax scheme can replicate the optimal equilibrium allocation even under private information.} Thus a banking contract with truth-telling constraints is considered to provide liquidity insurance efficiently under private information.

This paper provides two key findings. First, when the total supply of assets is insufficient to separate the types, a liquidity premium could arise in the price of illiquid assets even though illiquid assets are not useful for trade directly. It is because illiquid assets are useful for revealing the private type information. After the individual liquidity risk is realized, the optimal liquidity distribution of banks is to provide liquid assets to the impatient types. However, the patient types always have an incentive to mimic the impatient types. Thus banks are required to hold liquid or illiquid assets to inhibit the impatient types from withdrawing liquid assets. Thus when the supply of assets is insufficient to separate the types, the liquidity premium can arise in either liquid or illiquid asset prices.

Secondly, when the total supply of assets is insufficient to separate the types, a proportion of liquid assets, i.e. excess reserves, should be held in balance sheets of banks for the patient types to reveal their types. In this case although the government injects money in the markets by absorbing government bonds, the currency trade does not increase since the injected money would be just held as reserves to reveal the types. This liquidity trap equilibrium can exist when the truth-telling constraint binds to reveal the private information. Thus it could exist away from the zero-lower-bound in which the conventional liquidity trap equilibrium exists: a liquidity trap can exist when the rates of return in currency and government bonds are the same at the zero nominal interest rate.

The first finding is related to the literature about the liquidity premium on asset prices. Geromichalos et. al. (2007) and Lagos and Rocheteau (2008) show that the asset prices can have a liquidity premium when the assets are useful for exchange. This paper is different from these papers because illiquid assets are not useful for trade although there exists a liquidity premium on those assets when the supply of assets is insufficient. This usefulness of illiquid assets is also different from the reasons why the illiquid bonds are beneficial in Kocherlakota (2003) and Shi (2008). Kocherlakota (2003) shows that illiquid bonds can relax the cash-in-advance constraint as agents can trade assets after observing idiosyncratic shock. Shi (2008) shows that the welfare improves when government bonds are legally restricted for one type of trade. In those papers illiquid assets are useful because they are less liquid than liquid assets, but in this paper both liquid and illiquid assets can be useful to separate the types.

The second result is related to the literature on the implementation of monetary policy. Wallace (1981) studies the effectiveness of monetary policy, in particular open market operations, by applying the Modigliani-Miller theorem to the government liability structure. In this paper excess
reserves can exist in equilibrium when the truth-telling constraint binds so that monetary policy can be ineffective. These excess reserves are different from the ones in the liquidity trap as shown in Williamson (2012) because excess reserves are uniquely determined to reveal the types in this paper.

This paper is related to the literature on liquidity insurance. In their pioneering paper Diamond and Dybvig (1983) show that bank runs can exist as an equilibrium outcome when liquidity shocks are private information. Allen and Gale (1998) point out that the rates of return on long-term projects are critical to revealing the type information in the Diamond-Dybvig type model. In this paper private information is emphasized as a main friction. Moreover, there is no coordination failure of patient agents as shown in Diamond and Dybvig (1983) and no default by aggregate risk as in Allen and Gale (1998).

This paper builds on the previous banking models with an explicit asset trade. Freeman (1988) and Champ, Smith, and Williamson (1996) study banking and liquidity insurance with overlapping generation models. Recently, Bencivenga and Camera (2011) have used uncertainty in trading opportunity in the Lagos and Wright (2005) framework to introduce an insurance banking, but a standard debt contract is considered without individual incentive constraints as shown in their paper. Williamson (2012) constructs a Diamond-Dybvig type bank in the Rocheteau and Wright (2005) framework and shows that a liquidity trap can exist away from the Friedman rule.

The rest of the paper is organized as follows. Section 2 describes the environment. Section 3 offers a simple model with two different liquidity assets and compares the equilibrium allocations under perfect information and private information. Section 4 extends the model with government liabilities and monetary policy and analyzes in what circumstance monetary policy can be ineffective. Section 5 discusses the result to address implications, and Section 6 concludes.

2 Model

The basic structure of the model is based on Rocheteau and Wright (2005). Time $t = 0, 1, 2, \ldots$ is discrete in infinite horizon and each period is divided into two sub-periods - the Centralized Meeting ($CM$) followed by the Decentralized Meeting ($DM$). The population consists of two types of ex ante heterogeneous agents, buyers and sellers, who are infinitely lived with $[0, 1]$ continuum of each type. An individual buyer has preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t [-H_t + \theta_i u(x_t)]$$

where $H_t \in \mathbb{R}$ is labor supply in the $CM$, $x_t \in \mathbb{R}_+$ is consumption in the $DM$, and $0 < \beta < 1$. Assume that $u(\cdot)$ is strictly increasing, strictly concave, and twice continuously differentiable with
\( u'(0) = \infty, u(0) = 0, \) and \( -x \frac{u''(x)}{u(x)} < 1 \) for all \( x > 0. \)\(^2\) One variation is that each buyer is exposed to an idiosyncratic liquidity shock, \( \theta_i^t, \) which follows independent and identical distribution with two realizations \( \{1, \theta\} \) where \( \theta \in [0, 1) \) by types \( i = 1, 2. \) With probability \( \rho \in (0, 1) \) a buyer can be type 1 with \( \theta_1^t = 1 \) and otherwise a buyer is type 2 with \( \theta_2^t = \theta. \) There can be ex post heterogeneity in marginal utility across the buyers.\(^3\) Each seller has preferences as

\[
E_0 \sum_{t=0}^{\infty} \beta^t [X_t - h_t]
\]

where \( X_t \in \mathbb{R} \) is consumption in the CM, and \( h_t \in \mathbb{R}_+ \) is labor supply in the DM. Buyers can produce in the CM, but cannot produce in the DM while sellers can produce in the DM, but cannot produce in the CM. One unit of labor inputs can produce one unit of consumption goods in the CM and DM. It is assumed that the consumption goods are perfectly divisible and perishable.

There are two types of nominal government-issued assets. Fiat money trades at price \( \phi_t \) in terms of goods in the competitive market of period \( t \) CM. One-period maturity government bonds is an obligation to pay one unit of fiat money in the period \( t + 1 \) CM. The price of government bonds is \( z_t \) in terms of goods in the period \( t \) CM. There are also two types of real private assets - two divisible Lucas trees. Each tree is endowed to buyers in the initial period CM with a fixed unit supply. One tree, named as liquid tree, pays dividend \( y^l \) and trades at the price \( \psi^l_t \) in terms of goods in period \( t \) CM whereas the other tree, called as illiquid tree pays dividend \( y^i \) and trades at the price \( \psi^i_t \) in terms of goods in period \( t \) CM.\(^4\)

In the beginning of the CM, buyers and sellers meet together and debts are settled. Buyers receive lump-sum transfer or pay lump-sum tax and the the holders of the Lucas trees receive the realized dividends. A Walrasian market opens, assets and goods are traded competitively. In the DM each buyer meets each seller bilaterally and the terms of trade are determined by bargaining. The buyer make a take-it-or-leave-it offer to the seller in the meetings. I assume that all agents are anonymous and there is no public record-keeping technology in the CM and DM. Thus recognizable assets are essential for trade and all trade must be quid pro quo. Similar to Sanches and Williamson (2010), there are two types of meetings by the types of buyers in the DM. In a fraction \( \rho \) of DM meetings with type 1 buyers, fiat money and the liquid tree are the only assets recognized by sellers. In \( 1 - \rho \) fraction of DM meetings with type 2 buyers, the sellers can verify the entire portfolio held by buyers.\(^5\)

Limited commitment is assumed so that agents can run away in the next CM, but the

\(^2\)In the model asset demand is strictly increasing in rates of return when the coefficient of relative risk aversion is less than one. It guarantees to have at least one equilibrium exists.

\(^3\)This is different with Shi (2008) where illiquid bonds are beneficial because it can be used for higher marginal utility trade. It is also different with He, Huang and Wright (2008) where theft on money trade can lead higher marginal utility in deposit claims trade.

\(^4\)This Lucas tree represents private investment as described in Lagos and Rocheteau (2008), but since the supply is fixed the inefficiency is reflected to its price instead of quantity as shown in Lagos and Rocheteau (2008).

\(^5\)It is not contrary to no record-keeping technology assumption. Suppose sellers accept a deposit claim issued by
assets are seized and settled as a collateral. In sum type 1 buyers can trade only with fiat money or liquid tree and have a utility with $\theta^1_t = 1$ in the $DM$ whereas type 2 buyers can trade through credit arrangement with all their asset portfolio and have a utility with $\theta^2_t = \theta$.

Given idiosyncratic liquidity shock a banking arrangement arises endogenously to allocate liquid and illiquid assets by the type of buyers efficiently. Without a banking contract type 2 buyers could hold idle liquid assets while type 1 buyers could hold idle illiquid assets. All agents can propose a banking contract which provides liquidity insurance in a way of Diamond and Dybvig (1983). It is socially optimal that banks provide liquid assets including fiat money to type 1 buyers who move to $\rho$ proportion of meetings and provide illiquid assets including government bonds to type 2 buyers who move to $1 - \rho$ proportion of meetings. Banks observe the size of shock $\rho$ exactly, but banks cannot verify the type of buyers after realization. Thus one type of buyers can mimic the other type of buyers under private information.\(^6\) In this environment if the real value of liquid assets to type 1 buyers is greater than the value of illiquid assets to type 2 buyers then type 2 buyers will mimic type 1 buyers since liquid assets are also useful for trade in $1 - \rho$ proportion of meetings.

To support banking arrangement I assume that buyers can meet only a bank in the $CM$ after their liquidity shock is realized. If ex post asset-trading among buyers is allowed then banking contract is unraveled and collapsed as shown in Jacklin (1987).\(^7\)

Timing is as follows as shown in Figure 1. In the beginning of $CM$ deposit claims are paid and government-bonds holders can receive a unit of fiat money by redeeming a unit of government bonds. Then buyers receive lump-sum transfer(or pay lump-sum tax). All buyers provide labor and trade assets with sellers in a Walrasian market. Buyers deposit goods or assets into a banker with a banking contract. After liquidity shock is realized, buyers learn their types and $\rho$ buyers meet the banker to withdraw currency and liquid assets. In the $DM$ buyers meet sellers randomly in the bilateral meeting and make take-it-or-leave-it offers. In the next $CM$ $1 - \rho$ sellers can redeem deposit claims to the banker or sell them to buyers.

### 2.1 Government

In the model government consists of fiscal authority and monetary authority. Fiscal authority can enforce lump-sum tax or provide transfer to buyers in the $CM$ and issue government bonds and pay interests in the next $CM$. Monetary authority can issue fiat money and inject or absorb fiat money banks and backed by asset portfolio of buyers, and redeem it in the next $CM$.

\(^6\)In order to avoid bank runs created by sequential service I assume that buyers send a notification about their types to the bank simultaneously.

\(^7\)Note that a banking contract equilibrium provides higher welfare than assets-trading market equilibrium. It is because a bank contract can provide resources to high marginal utility agents as much as it can regardless of prices.
by exchanging money with government bonds in the asset market, i.e. open-market-operations. I assume that private assets are not eligible to be an object for OMOs. Thus after government bonds are issued and fiat money is injected by open-market-operations at \( t = 0 \) and the revenue of issuing government bonds and fiat money is transferred to buyers. Then outstanding fiat money and government bonds can be supported by tax or transfer over time. So the consolidated government budget constraints are described as

\[
\phi_0(M_0 + z_0B_0) = \tau_0 = V
\]

and

\[
\phi_t\{M_t - M_{t-1} + z_tB_t - B_{t-1}\} = \tau_t, \ t = 1, 2, 3, \ldots
\]

where \( M_t \) and \( B_t \) denote the nominal quantities of fiat money and government bonds held by private sector in the CM at time \( t \), respectively. \( \tau_t \) denote the real value of the lump-sum transfer from each buyer to the fiscal authority in the CM at period \( t \). I assume that the fiscal authority keeps the total value of the outstanding consolidated government debt as a constant \( V \) after it is transferred with an exogenously fixed amount at \( t = 0 \). Thus in every period to maintain the real value of outstanding consolidated government debt, the real term of lump-sum transfer \( \tau_t \) is derived passively from

\[
\tau_t = \left( V_t - \frac{\phi_t}{\phi_{t-1}}V_{t-1} \right) + \frac{\phi_t}{\phi_{t-1}}(1 - z_{t-1})\phi_{t-1}B_{t-1}, \ t = 1, 2, 3, \ldots
\]

Note that the lump-sum transfer consists of seigniorage from inflation and real interest payment for government bonds. The fixed real value of consolidated government debt assumption allows us to separate monetary policy, specifically OMOs, from fiscal policy. Moreover, when \( V \) is assumed as small enough, assets can be insufficient to support the optimal level of consumption.

3 Competitive Equilibrium with Liquid and Illiquid Assets

In this section to emphasize private information friction I assume that illiquid assets are not useful for trade at all with \( \theta = 0 \). Also, in order to make a clear point I exclude the government liabilities by assuming \( V = 0 \). So we have only two different Lucas trees in the model. One is useful for trade with price \( \psi^l_t \) and dividend \( y^l_t \) while the other cannot be used in transactions at all with price \( \psi^i_t \) and dividend \( y^i_t \). In the following subsections I consider each perfect information and private information case and compare the equilibrium allocations to understand the reason why the liquidity insurance can be restricted by the private information. Under private information banks suggest two type-dependent consumption offers \( \{(x^l_1, x^l_1), (x^l_2, x^l_2)\} \) for type 1 and type 2 buyers to reveal their types.
Note that superscripts denote type of assets between liquid and illiquid while subscripts denote type \( j \) of buyers: \( x^j_{1t}, x^j_{2t} \) represents the consumption level of type \( j \) buyers with liquid and illiquid assets. Under perfect competition a representative bank suggests a banking contract to maximize buyers’ ex ante expected value. In equilibrium the bank solves the following generalized problem in the CM of period \( t \):

\[
\begin{align*}
Max & \quad d_t, a^l_t, a^i_t, x^l_{1t}, x^i_{1t}, x^l_{2t}, x^i_{2t} - d_t + \rho \{ u(x^1_{1t}) + x^i_{1t} \} + (1 - \rho) \{ x^l_{2t} + x^i_{2t} \} \\
\text{subject to a participation constraint of the bank,} & \quad d_t - \psi^l_t a^l_t - \psi^i_t a^i_t + \{ \beta(\psi^l_{t+1} + y^l ) a^l_t - \rho x^l_{1t} - (1 - \rho)x^l_{2t} \} + \{ \beta(\psi^i_{t+1} + y^i ) a^i_t - \rho x^i_{1t} - (1 - \rho)x^i_{2t} \} \geq 0 \\
\text{and incentive constraints of the bank,} & \quad \beta(\psi^l_{t+1} + y^l ) a^l_t - \rho x^l_{1t} \geq 0 \\
& \quad \beta(\psi^l_{t+1} + y^l ) a^l_t + \beta(\psi^i_{t+1} + y^i ) a^i_t - \rho x^i_{1t} - (1 - \rho)x^i_{2t} \geq 0 \\
\text{and truth-telling constraints,} & \quad u(x^l_{1t}) + x^i_{1t} \geq u(x^l_{2t}) + x^i_{2t} \\
& \quad x^l_{2t} + x^i_{2t} \geq x^l_{1t} + x^i_{1t} \\
\text{and non-negative constraints,} & \quad d_t, a^l_t, a^i_t, x^l_{1t}, x^i_{1t}, x^l_{2t}, x^i_{2t} \geq 0
\end{align*}
\]

All quantities in equations (1)-(7) are expressed in units of the CM good in time \( t \). The problem (1) subject to constraints (2)-(7) states that a banking contract is chosen in equilibrium to maximize the expected utility of the representative buyer subject to the participation constraint for the banks (2) and liquid asset constraint (3) and collateral constraint (4) and individual incentive constraints by types (5)-(6) and non-negativity constraints (7). In (1)-(7) \( d_t \) denotes deposit of buyers, \( a^l_t, a^i_t \) denote demand for liquid and illiquid asset holdings of banks, \( \psi^l_t, \psi^i_t \) denote the prices of liquid and illiquid assets in the CM, respectively. The quantity on the left side of (2) is the net payoff for banks. In the CM of time \( t \) the banks receive \( d_t \) deposits and invest in liquid and illiquid assets with market prices then the banks pay \( x^l_{j_t} \) to each type buyer before the DM and pay \( x^i_{j_t} \) to the holders of deposit claims in the following CM. The participation constraint (2) implies that when
deposit claims are paid off, the net payoff for the banks must be greater than zero. The liquid asset constraint (3) implies that type 1 buyers can withdraw liquid assets by the limit of liquid asset holdings. The collateral constraint (4) implies that the liquid and illiquid assets can be seized when the bank decides to abscond. Individual incentive constraint (5)-(6) represent that each type of buyer weakly prefer an offer for own type to the offer for other type after type shock is realized. Note that type 1 buyers can also consume with illiquid assets in the next CM.

3.1 Perfect Information

For a benchmark I consider competitive equilibrium with perfect information. In case of perfect information banks know the buyer’s type exactly after the liquidity shock is realized. In equilibrium ex post banks will provide all of liquid assets to type 1 buyers who only can trade in the DM. Since illiquid assets are useless for trade banks will not hold these assets as long as the real return of the illiquid asset are less than time preference. If the real return of the illiquid asset is same as time preference then banks can hold these illiquid assets and provide them to type 1 or 2 buyers, but it is irrelevant since both type buyers have linear utility function for illiquid assets. Thus without loss of generality I assume that banks do not hold illiquid assets, i.e. $x_{1t} = x_{2t} = 0$, with perfect information. Moreover, truth-telling constraints are unnecessary. Thus given price $\psi_t^l$, a representative bank solves the reduced maximization problem in the CM of period $t$:

$$Max_{d_t, a_t^l, x_{1t}^l} - d_t + \rho u(x_{1t}^l) \tag{8}$$

subject to the participation constraint,

$$d_t - \psi_t^l a_t^l + \{\beta(\psi_{t+1}^l + y^l)a_t^l - \rho x_{1t}^l\} \geq 0 \tag{9}$$

and liquid asset constraint,

$$\beta(\psi_{t+1}^l + y^l)a_t^l - \rho x_{1t}^l \geq 0 \tag{10}$$

and non-negative constraints,

$$d_t, a_t^l, x_{1t}^l \geq 0 \tag{11}$$

By plugging (9) into (8) we have the first-order conditions by $a_t^l, x_{1t}^l$,

$$\psi_t^l = \beta(\psi_{t+1}^l + y^l)(1 + \lambda_t) \tag{12}$$

$$u'(x_{1t}^l) - 1 = \lambda_t \tag{13}$$
where $\lambda_t$ is a multiplier associated with liquid asset constraint (10). The first-order conditions (12) and (13) can be reduced to

$$\psi^l_t = \beta(\psi^l_{t+1} + y^l)u'(x^l_{1t})$$

(14)

In equilibrium asset market clear in the CM and a representative bank holds all the liquid asset in its portfolio for $t = 0, 1, 2, \ldots$. The supply of liquid asset is equal to the demand of banks as

$$a^l_t = 1.$$  

(15)

**Definition 1**: Given $(\rho, y^l, y^i)$ a stationary competitive equilibrium under perfect information consists of quantity $x^l_{1t}$ and price $\psi^l_t$ and multiplier $\lambda$ which solves equations (10), (14), (15).

In what follows I focus on stationary equilibrium allocations without time scripts on variables. There are two equilibrium cases following the value of $y$.

**Case (i)** Suppose that the liquid asset constraint (10) does not bind. That means, in equilibrium the first-best consumption level, $x^*$ where $u'(x^*) = 1$, is achieved for type 1 buyers. From the first-order condition (14), the asset price is the same as its fundamental value: $\psi^l = \psi^l_f$ holds where $\psi^l_f \equiv \frac{\beta y^l}{1-\beta}$. Note that this case of equilibrium is supported by $y^l \geq \frac{(1-\beta)}{\beta} \rho x^*$ from (10).

**Case (ii)** Suppose that the liquid asset constraint (10) binds with $y^l < \frac{(1-\beta)}{\beta} \rho x^*$. Then the equilibrium allocation $(x^l_{1}, \psi^l)$ is uniquely determined from (10) and (14) since $\psi^l$ is strictly increasing in $x^l_{1}$ from (10) while $\psi^l$ is strictly decreasing in $x^l_{1}$ from (14). Note that the consumption level is less than its optimal level, $x^l_{1} < x^*$ and the asset price is greater than its fundamental value, $\psi^l > \psi^l_f$ in equilibrium. Liquidity premium, the difference between the asset price and its fundamental value is strictly positive because of liquid asset shortage. The price of illiquid asset is same as its fundamental value as $\psi^i = \psi^i_f$ where $\psi^i_f \equiv \frac{\beta y^i}{1-\beta}$, if it is traded in the market.

These two cases can be described in Figure 2. If the supply of liquid asset is large enough with $y^l \geq \frac{(1-\beta)}{\beta} \rho x^*$ then we have the case 1 equilibrium with the first-best allocation. If the supply of liquid asset is low with $y^l < \frac{(1-\beta)}{\beta} \rho x^*$ then we would have this case 2 equilibrium.

[Figure 2 here]

### 3.2 Private Information

In case of private information as described above, banks solve the original maximization problem (1)-(7). To simplify the problem I use some lemmas here.

**Lemma 1.** (Single Crossing Property) In equilibrium with $x^l_{1t} \in [0, x^*)$, both truth-telling constraints do not bind simultaneously.
proof If both truth-telling constraints (5) and (6) bind then \( u(x_{1t}^l) - x_{1t}^l = u(x_{2t}^l) - x_{2t}^l \). Since 
\( u(x) - x \) is strictly increasing in \( x \in [0, x^*] \), \( x_{1t}^l = x_{2t}^l \) and \( x_{1t}^i = x_{2t}^i \). Note that \( x_{1t}^i = x_{2t}^i > 0 \) for \( y > 0 \) in equilibrium. For \( x_{1t}^l = x_{2t}^l < x^* \) the expected value of buyers can increase by transferring liquid assets from type 2 buyers to type 1 buyers and transferring the same amount of illiquid assets from type 1 buyers to type 2 buyers. Contradiction. 

Lemma 2. In equilibrium with \( x_{1t}^l \in [0, x^*) \), the truth-telling constraint for type 1 buyers does not bind.

proof Suppose that the constraint (5) binds while the constraint (6) does not bind. If \( x_{2t}^i > 0 \) then the expected value of buyers can increase by transferring liquid assets from type 2 buyers to type 1 buyers. If \( x_{2t}^i = 0 \) then the expected value of buyers are indifferent when illiquid assets are transferred from type 2 buyers to type 1 buyers that means (6) does not bind. Contradiction.

In equilibrium the truth-telling constraint for type 1 buyers (5) does not bind. It is because banks allocate resources to type 1 buyers who have higher marginal utility as much as possible to maximize the expected utility for agents. Thus the incentive constraint for type 2 buyers always binds. In Figure 3, indifference curve of type 1 buyers intersects the indifference curve of type 2 buyers at \( x_{1t}^l \). To keep utility for type 2 buyers \( x_{2t}^i \) is required. Since (5) does not bind, there is no difference between \( x_{2t}^l \) and \( x_{2t}^i \) in the problem so that they can be merged as \( x_{2t} \). Note that type-dependent contract is non-linear in general, but in this case deposit contract is linear as standard deposit contract because quasi-linear utility is adopted.

[Figure 3 here]

Lemma 3. In equilibrium with \( x_{1t}^l \in [0, x^*) \), \( x_{1t}^l = 0 \).

proof Suppose that \( x_{1t}^l > 0 \) in equilibrium. If the truth-telling constraint for type 2 buyers (6) binds then transferring illiquid assets to type 2 buyers can relax (6). If (6) does not bind then the expected value of buyers is indifferent. Thus there is no reason to have \( x_{1t}^l > 0 \) in equilibrium.

Illiquid assets for type 1 buyers are unnecessary because neither it is used for trade nor it overcomes private information friction in (12). Since \( x_{1t}^l = 0 \) in equilibrium we can rename \( x_{1t}^l \) as \( x_{1t} \). Without the truth-telling constraint for type 1 buyers (5) and with choice variable \( x_{1t} \) and \( x_{2t} \), we have the first-order conditions by \( a_{1t}, a_{2t}, x_{1t}, x_{2t}, \)

\[
\psi_{ti} = \beta(\psi_{t+1}^l + y_{ti}^l)(1 + \lambda_{1t} + \lambda_{2t})
\] (16)
\[ \psi_i^t = \beta(\psi_{i+1}^t + y^i)(1 + \lambda_2t) \]  
(17)

\[ \rho u'(x_{1t}) - \rho - \rho \lambda_{1t} - \lambda_{3t} = 0 \]  
(18)

\[ (1 - \rho) - (1 - \rho) - (1 - \rho)\lambda_{2t} + \lambda_{3t} = 0 \]  
(19)

where \( \lambda_{1t}, \lambda_{2t} \) and \( \lambda_{3t} \) denote each multiplier associated with the constraints (3), (4) and (6). In equilibrium asset markets clear in the CM and a representative bank holds all the liquid and illiquid assets in its portfolio for \( t = 0, 1, 2, \ldots \). The supply of liquid and illiquid asset is equal to its demand from banks, respectively, as shown in (20).

\[ a^t = a^i = 1 \]  
(20)

**Definition 2 :** Given \((\rho, y^l, y^i)\) a stationary competitive equilibrium under private information consists of quantity \( x_1, x_2 \) and price \( \psi^l, \psi^i \) and multiplier \( \lambda_{1t}, \lambda_{2t}, \lambda_{3t} \) which solves equations (3)-(4),(6),(16)-(20).

I focus on stationary equilibrium allocations without time scripts on variables. Note that the collateral constraint (4) and the truth-telling constraint (6) either binds or does not bind together from (19) because \( x_2 > 0 \) in equilibrium with \( y^i > 0 \). If the truth-telling constraint (6) binds with \( \lambda_3 > 0 \) then the liquid asset constraint (3) is relaxed with \( \lambda_1 = 0 \) because only one of them can restrict \( x_1 \in [0, x^*] \) in equilibrium. In sum under private information there are three equilibrium cases. As discussed in perfect information subsection we have case (i) and (ii) equilibrium and additionally a new equilibrium, case (iii) equilibrium, in which the truth-telling constraint (6) binds and the liquid asset constraint (3) is relaxed.

**Case (i)** When the constraints (3), (4), (6) do not bind we have

\[ \frac{\psi^l}{\beta(\psi^l + y^l)} = \frac{\psi^i}{\beta(\psi^i + y^i)} = 1 \]  
(21)

and

\[ 1 = u'(x_1) \]  
(22)

from the first-order conditions (16)-(18). Since the constraints do not bind, the optimal level of consumption is achieved, \( x_1 = x^* \), for type 1 buyers and the asset prices are the same as their fundamental values, \( \psi^l = \psi^i = \psi_f \) from (21)-(22). The equilibrium is supported by a region with \( \psi_f \geq \rho x^* \) and \( \psi_f + \psi_f^i \geq x^* \) from (3)-(4), (20). Note that this first-best equilibrium allocation
is the same as one in perfect information case (i). One interesting point is that this first-best equilibrium allocation is feasible even when there exists a degree of private information as long as liquid and illiquid assets are plentiful in the economy.

**Case (ii)**  When the liquid asset constraint (3) binds only, we have

\[
\frac{\psi^i}{\beta(\psi^i + y^i)} = 1
\]

and

\[
\frac{\psi^l}{\beta(\psi^l + y^l)} = u'(x_1)
\]

from (16)-(18). The binding constraint (3) with asset market clear condition (20) can be reduced into

\[
\beta(\psi^l + y^l) = \rho x_1
\]

Then \((x_1, \psi^l)\) are uniquely determined from (24) and (25). In equilibrium we have \(x_1 < x^*, \psi^l > \psi^l_f\), \(\psi^i = \psi^i_f\) and \(x_1 \leq x_2\). Note that a liquidity premium arises in the price of the liquid asset since the inefficiency is caused by the scarcity of liquid asset as described in Champ, Smith and Williamson (1996). On the other hand, the price of illiquid asset keeps at its fundamental value because those illiquid assets are already plentiful. The equilibrium is supported by a region which satisfies with \(\psi^l_f < \rho x^*\) and \(y^l \geq \frac{\rho}{1-\rho} y^i\). If \(y^i\) is too low then there exists a threshold point in which the truth-telling constraint (6) starts to bind while the liquid asset constraint (3) is just relaxed. In this threshold point we have \(\beta(\psi^l + y^l) = \rho x_1, \beta(\psi^i + y^i) = (1-\rho)x_2\), \(x_2 = x_1\) from (3), (4), (6). We also have \(\frac{\psi^i}{\beta(\psi^i + y^i)} = \frac{\psi^l}{\beta(\psi^l + y^l)}\) from (16)-(17) since the liquid asset constraint (3) is just slack at the point. Note that those conditions can be reduced into \(y^l = \frac{\rho}{1-\rho} y^i\) which is a threshold borderline between case (ii) and case (iii) equilibrium. Note that this case (ii) equilibrium is also the same as the case 2 equilibrium under perfect information.

**Case (iii)**  When the collateral constraint (4) and the truth-telling constraint (6) bind, we have

\[
\frac{\psi^l}{\beta(\psi^l + y^l)} = \frac{\psi^i}{\beta(\psi^i + y^i)} = \frac{\rho u'(x) + 1 - \rho}{1 - \rho}
\]

and

\[
\beta(\psi^l + y^l) + \beta(\psi^i + y^i) = x
\]

from (16)-(20) where \(x \equiv x_1 = x_2\) is denoted. Then \((x, \psi^l, \psi^i)\) are uniquely determined from the equilibrium condition (26)-(27). In equilibrium we have \(x < x^*, \psi^l > \psi^l_f, \psi^i > \psi^i_f\). Note that a
liquidity premium arises in the both prices of liquid and illiquid assets although the liquid asset constraint does not bind in equilibrium. It is because both liquid and illiquid assets are scarce to reveal the private information in equilibrium. In equilibrium a proportion of liquid assets must be provided to type 2 buyers so that rates of return in liquid asset and illiquid asset are same in equilibrium. Note that a liquidity premium arises although illiquid assets are not useful for trade at all.

Three regions of equilibrium under private information are described in Figure 4. Notice that the first-best equilibrium allocation appears when liquid assets are plentiful and are supported by sufficient illiquid assets. The case (ii) equilibrium allocation is shown when liquid assets are scarce although those liquid assets are supported by illiquid assets enough. The case (iii) equilibrium allocation arises when the types are hardly revealed because of the scarcity of illiquid assets.

4 Monetary Equilibrium

In this section I verify how this private information restricts the implementation of monetary policy, specifically open-market-operations. We have money and government bonds in the model with \( V > 0 \), but do not have private liquid assets with \( y^l = 0 \). Note that private liquid assets are irrelevant because we focus on the case in which illiquid assets are scarce. Also type 2 buyers can consume through credit arrangement. However, the marginal utility in credit arrangement is less than the marginal utility in currency trade with \( \theta \in (0, 1) \). This assumption represents that credit arrangement trade is less preferable than currency trade in a view of social welfare. One rationalization of this assumption is a social cost of operating payment and settlement for credit arrangement. Note that this assumption will provide an incentive for banks to increase currency exchange more than credit arrangement for social optimality. Thus this assumption lets the truth-telling constraint bind when the bank maximizes the ex ante expected value of buyers. Note that the model can be reduced to the baseline model of Williamson (2014) if type information is perfect and \( \theta = 1 \). A representative bank solves the following modified problem in the CM of period \( t \):

\[
\max_{d_t, m_t, z_t, a_t, x_{1t}, x_{2t}} - d_t + \rho u(x_{1t}) + (1 - \rho) \theta u(x_{2t})
\]

subject to the participation constraint,

\[
d_t - m_t - z_t b_t - \psi^i_a^i_t + \left\{ \beta \phi_{t+1} m_t - \rho x_{1t} \right\} + \left\{ \beta \phi_{t+1} b_t + \beta (\psi^i_{t+1} + y^l) a^i_t - (1 - \rho) x_{2t} \right\} \geq 0
\]

and the cash constraint,
The problem (28) subject to the constraints (29)-(33) states that a banking contract is chosen in equilibrium to maximize expected utility of the buyers subject to participation constraint (29) in which banks receive a non-negative profit by providing the contract and cash constraint for type 1 buyers (30) and collateral constraint for type 2 buyers (31) and the incentive constraint for type 2 buyers (32) and non-negative constraints (33). I omitted the incentive constraint for type 1 buyers because it does not bind in equilibrium as shown in Lemma 1 and 2.8 In (28)-(33), \(d_t\) denotes deposit of buyers, \(a_t\) denote demand for illiquid asset holdings of banks, \(\psi_t\) denote the prices of illiquid assets in the CM, \(m_t\) and \(b_t\) denote the quantities of money and government bonds in terms of the CM good in period \(t\) held by banks and \(x_{jt}\) denote the consumption of type \(j\) agents at time \(t\) CM for \(j \in \{1, 2\}\). From now on I focus on stationary equilibrium where \(\frac{\phi_{t+1}}{\phi_t} = \frac{1}{\mu}\) holds for all \(t\), and \(\mu\) is the gross inflation rate. Note that nominal interest rate of government bonds cannot be negative, i.e. \(z_t \leq 1\), in equilibrium by its feasibility assumption. I assume that monetary authority sets the inflation rate target and implement OMOs to achieve its goal.

From the maximization problem the first-order conditions by \(m_t, b_t, a_t, x_{1t}, x_{2t}\) can be described as follows.

\[
\frac{\mu}{\beta} = 1 + \lambda_1 + \lambda_2 \quad (34)
\]

\[
z \frac{\mu}{\beta} = \frac{\psi}{\beta(\psi + y)} = 1 + \lambda_2 \quad (35)
\]

\[
u'(x_1) - \frac{\lambda_3}{\rho} = 1 + \lambda_1 + \lambda_2 \quad (36)
\]

8Both individual incentive constraints do not change although type 2 buyers can trade since liquid and illiquid assets are used as same for collateral.
\[ \theta u'(x_2) + \frac{\lambda_3}{1 - \rho} = 1 + \lambda_2 \] (37)

where \( \lambda_1 \) to \( \lambda_3 \) denote each multiplier associated with the constraints (30)-(32). In equilibrium asset markets clear in the CM with

\[ a^i = 1 \] (38)

\[ m = \phi M \] (39)

\[ b = \phi B \] (40)

Since the supply of government assets are restricted by the consolidated government debt limit \( V \) we have

\[ m + zb \leq V \] (41)

**Definition 3:** Given \( (\rho, V, y) \) and the inflation rate target \( \mu \), a stationary monetary equilibrium consists of quantities \( (x_1, x_2) \) and prices \( (z, \psi) \) and multipliers \( (\lambda_1, \lambda_2, \lambda_3) \) which solve equations (30)-(32), (34)-(37), (41).

Since quasi-linear utility is adopted the real return of assets such as fiat money, \( \mu = \frac{\phi_t - 1}{\phi_t} \), and government bonds, \( \frac{1}{z\mu} \), illiquid Lucas tree, \( \frac{\psi^i + y^i}{\psi^i} \), cannot exceed the rate of time preference, \( \frac{1}{\beta} \). The rate of returns in government bonds and Lucas tree are same in equilibrium because there is no credit risk in Lucas tree. Nominal interest rate of government bonds cannot be negative, i.e. \( z \leq 1 \), by its feasibility assumption. Then we have no arbitrage condition in equilibrium,

\[ \frac{1}{\mu} \leq r = \frac{1}{z\mu} = \frac{\psi^i + y^i}{\psi^i} \leq \frac{1}{\beta}. \] (42)

Note that if the truth-telling constraint (32) binds then cash constraint (30) is relaxed because only one of them can restrict \( x_1 \in [0, x^*] \) in equilibrium. Moreover, when the truth-telling constraint (32) binds collateral constraint (31) is required to bind, otherwise the truth-telling constraint (32) does not bind with \( x_2 = x_2^* \). Hence when the truth-telling constraint (32) binds we have only one case of equilibrium with \( \lambda_1 = 0, \lambda_2 > 0, \lambda_3 > 0 \). On the other hand, if the truth-telling constraint (32) does not bind then we have four cases of equilibrium with combination of \( \lambda_1 \) and \( \lambda_2 \). However, the case constraint (30) cannot be relaxed alone because the liquid assets can be also useful for credit arrangement. Thus we have totally four equilibrium cases.
(1) **Friedman rule Equilibrium** When all of the constraints (30)-(32) do not bind with $\lambda_1 = \lambda_2 = \lambda_3 = 0$, we have

$$\frac{\mu}{\beta} = z \frac{\mu}{\beta} = \frac{\psi^i}{\beta (\psi^i + y^i)} = 1 \quad (43)$$

$$1 = u'(x_1) = \theta u'(x_2) \quad (44)$$

from the first-order conditions (34)-(37). Note that $x_1 = x^*_1, x_2 = x^*_2$ and $\psi^i = \psi^j$ where $u'(x^*_1) = 1$, $\theta u'(x^*_2) = 1$, $\psi^j = \frac{\beta y^j}{1 - \beta}$ in equilibrium. Moreover, the rates of return in both liquid and illiquid assets are the same as the inverse of time preference, $\frac{1}{\mu} = r = \frac{1}{\beta}$ where $r \equiv \frac{1}{\beta} = \frac{\psi^i + y^i}{\psi^i}$ from (43)-(44) in equilibrium. Thus in this case the Friedman rule, $\mu = \beta$, is feasible so that the first-best allocation is achieved. The equilibrium is supported by a region with $V \geq \rho x^*_1$ and $V + \psi^i \geq \rho x^*_1 + (1 - \rho) x^*_2$ from the equations (30)-(31), (38), (41). Note that this case is the same as the case (i) in competitive equilibrium in the previous section.

(2) **Currency-shortage Equilibrium** Suppose that the Friedman rule equilibrium is infeasible with $V < \rho x^*_1$, while $\psi^j \geq (1 - \rho)x^*_2$ is valid. With $\lambda_1 > 0, \lambda_2 = \lambda_3 = 0$ we have the first-order conditions,

$$\frac{\mu}{\beta} = u'(x_1) \quad (45)$$

$$z \frac{\mu}{\beta} = \frac{\psi^i}{\beta (\psi^i + y^i)} = 1 = \theta u'(x_2) \quad (46)$$

in which $x_2 = x^*_2$ and $\psi^i = \psi^j$. Binding cash constraint (30) is transformed into $\rho x^*_1 u'(x_1) = V$. Thus $x_1 < x^*_1$ is fixed in equilibrium. Then in a region of $x_1 \in (0, x^*_1)$ we have $x_1 < x^*_2$ and $\frac{1}{\mu} < r = \frac{1}{\beta}$ in equilibrium. Define this case as **Currency-shortage Equilibrium**. In this case real interest rate of illiquid asset is fixed and a liquidity premium arises in the price of money since only money is scarce. Open market operations are ineffective in real allocations since real interest rate of illiquid assets and $x_1$ are fixed in equilibrium. Note that this case is the same as the case (ii) in competitive equilibrium in the previous section.

(3) **Asset-shortage Equilibrium** Suppose that the Friedman rule equilibrium is infeasible with $V + \psi^j < \rho x^*_1 + (1 - \rho)x^*_2$, but $V \geq \rho x^*_1$ is still valid. Moreover, assume that $\lambda_3 = 0$. Then we have the first-order conditions,

$$\frac{\mu}{\beta} = u'(x_1) \quad (47)$$

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\[ z \frac{\mu}{\beta} = \frac{\psi^i}{\beta(\psi^i + y^i)} = \theta u'(x_2) \]  

(48)

from (34)-(37). Binding constraints (30) and (31) with asset market clearing condition (38) and government budget constraint (41) can be reduced into

\[ \rho x_1 u'(x_1) + (1 - \rho) \theta x_2 u'(x_2) = V + \frac{\beta y^i \theta u'(x_2)}{1 - \theta u'(x_2)} \]  

(49)

Given the price of government bonds \( z \), \( x_1 \) and \( x_2 \) are positively related in (47) and (48) while \( x_1 \) and \( x_2 \) is negatively related in (49). Thus there exists a unique equilibrium allocation \((x_1, x_2)\) as shown in Figure 5. Let me briefly describe threshold points as shown in Figure 5. There is a threshold point \( \hat{x}_1 \) at \( x_2 = x_2^* \) in which the collateral constraint starts to bind. There is another point \( \tilde{x}_1 \) where the equilibrium allocation is determined with the zero nominal interest rate, \( z = 1 \). Through open market operations, the monetary authority can inject money and absorb government bonds in the market. By conducting this procedure the currency trade \( x_1 \) increases whereas the credit arrangement \( x_2 \) decreases. Thus the monetary authority can choose an equilibrium allocation in \( x_1 \in (\hat{x}_1, \tilde{x}_1] \) by choosing the price of government bonds \( z \) in equilibrium. However, in this private information case the truth-telling constraint matters when \( x_1 \) becomes greater than \( x_2 \). Thus this asset-shortage equilibrium can exist only in \( x_1 \in (\hat{x}_1, \tilde{x}_1] \) with \( z \leq \theta \).

(4) Liquidity-trap Equilibrium  

Given the Friedman rule equilibrium is infeasible with \( V + \psi^i f < \rho x_1^* + (1 - \rho)x_2^* \) and \( V \geq \rho x_1^* \), suppose that truth-telling constraint (32) binds with \( \lambda_3 > 0 \). As discussed when (32) binds we have only one case of equilibrium with \( \lambda_1 = 0, \lambda_2 > 0, \lambda_3 > 0 \). The first-order conditions are reduced into

\[ \mu \beta = z \beta = \psi^i (x_1^*) - \frac{\lambda_3}{\rho} = 1 + \lambda_2 \]  

(50)

\[ \theta u'(x_2) + \frac{\lambda_3}{1 - \rho} = 1 + \lambda_2 \]  

(51)

from (34)-(37). Since the truth-telling constraint binds, we have \( x \equiv x_1 = x_2 \) in equilibrium. Thus (50) and (51) can be reduced into

\[ u'(x) - \frac{\lambda_3}{\rho} = \theta u'(x) + \frac{\lambda_3}{1 - \rho} \]  

(52)

Then binding constraints (31)-(32) with asset market clearing condition (38) and government budget constraint (41) can be reduced into

\[ \rho u'(x) + (1 - \rho) \theta u'(x) = V + \frac{\beta y^i \theta u'(x)}{1 - \theta u'(x)} \]  

(53)

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The allocation \( x \) is determined at \( x = \hat{x}_1 \) in which government budget constraint intersects with 45 degree line as shown in Figure 5. Note that \( \hat{x}_1 \) is fixed and \( \frac{1}{\mu} = r < \frac{1}{\beta} \) holds with \( z = 1 \) in equilibrium. Thus in the *Liquidity-trap equilibrium* rates of return in money and illiquid assets are same and open market operations are no longer effective. Note that the feasibility condition (53) is similar to (49) with \( x_1 = x_2 \). However, the consumption level \( x \) in (53) is greater than the consumption level \( x_1 = x_2 \) in (49) with \( z = \theta \) because liquid assets also have a liquidity premium in the liquidity trap equilibrium.

[Figure 5 here]

### 4.1 Liquidity Trap and Excess Reserves

In this subsection let me elaborate the implementation of monetary policy in the *Asset-shortage Equilibrium* and *Liquidity-trap equilibrium*. Given the scarcity of illiquid assets with \( V + \psi_f^i < \rho x_1^s + (1 - \rho)x_2^s \) and \( V \geq \rho x_1^s \), there exist two different regions in the equilibrium under private information. In a region of \( x_1 \in (\hat{x}_1, \tilde{x}_1] \) we have \( x_1 < x_1^s \), \( x_2 < x_2^s \) and \( \frac{1}{\mu} < r < \frac{1}{\beta} \) in the *Asset-shortage Equilibrium*. In equilibrium the monetary authority can choose the equilibrium allocation along with the feasibility condition (49) by exchanging outside currency and government bonds in the market. For example, injecting money and absorbing government bonds decreases the nominal interest rate, \( \frac{1}{z} - 1 \), and currency trade \( x_1 \) increases whereas credit arrangement \( x_2 \) decreases. On the other hand, in a point of \( x_1 = \hat{x}_1 \) we have \( x_1 < x_1^s \), \( x_2 < x_2^s \) and \( \frac{1}{\mu} = r < \frac{1}{\beta} \) in the *Liquidity-trap Equilibrium* as described.

Thus if the type information is public, then the equilibrium allocation \( x_1 = \tilde{x}_1 \) with \( z = 1 \) is feasible. Note that in this equilibrium we have \( x_2 < x_1 \) and \( \frac{1}{\mu} = r < \frac{1}{\beta} \). Let’s define this equilibrium case as *Zero-lower-bound (ZLB) Equilibrium*. In the *Zero-lower-bound equilibrium* rates of return in money and illiquid assets are also same as shown in *Liquidity-trap equilibrium*. Thus open market operations are also ineffective in real allocations. Injecting money just increases the amount of excess reserves and the nominal interest rate is zero in equilibrium. However, the equilibrium allocations are different between *Zero-lower-bound equilibrium* and *Liquidity-trap equilibrium*. Moreover, the reasons for the ineffective monetary policy are different. In the *Zero-lower-bound equilibrium*, monetary policy is ineffective because the rates of return are set as same for both liquid and illiquid assets. But in the *Liquidity-trap equilibrium* monetary policy is ineffective because the excess reserves are required to separate the types under private information. Thus the same rates of return on liquid and illiquid assets with zero nominal interest rate is a consequence of equilibrium allocation instead of choice of monetary authority.

**Proposition 1.** Given \( \theta < 1 \), there exists a unique *Liquidity-trap equilibrium* away from the *Zero-lower-bound equilibrium* under private information.
**Proof.** In Liquidity-trap equilibrium the allocation \( x = x_1 = x_2 \) which satisfies (53). It is unique since the left side of (53) is strictly increasing in \( x \) while the right side of (5) is strictly decreasing in \( x \). The allocation \( x = x_1 = x_2 \) which satisfies (53) is different from the allocation in the Zero-lower-bound equilibrium because \( x_1 > x_2 \) holds in the Zero-lower-bound equilibrium.

Note that if \( \theta = 1 \) is assumed, Liquidity-trap equilibrium overlaps with Zero-lower-bound equilibrium because the truth-telling constraint does not bind even at the zero nominal interest rate.

It is hard to differentiate the existence of Liquidity-trap equilibrium from Zero-lower-bound equilibrium in reality. However, if there exists a cost of operating credit arrangement or inefficiency in credit arrangement such as haircut then there could exist a jump from \( z = \theta \) to \( z = 1 \) in equilibrium. Figure 6, which is replicated from Orphanides (2004), describes a movement of nominal interest rates along with excess reserves in the period of Great Depression. It is shown that there exists a volatile movement in nominal interest rates with excess reserves in a period of Great Depression. This implies at least that the monetary authority could lose its control on nominal interest rates in a neighborhood of zero lower bound.

### 4.2 Private Assets

In this subsection I discuss whether the scarcity of private assets can influence in the equilibrium allocations when incentive constraint binds. If there is no private illiquid assets, \( y^i = 0 \), then the equilibrium conditions (49) and (53) are transformed into

\[
\rho x_1 u'(x_1) + (1 - \rho)\theta x_2 u'(x_2) = V \tag{54}
\]

and

\[
\rho xu'(x) + (1 - \rho)\theta xu'(x) = V, \tag{55}
\]

respectively. Then since (54) and (55) are same when the truth-telling constraint binds with \( x_1 = x_2 \), the equilibrium allocation does not change as excess reserves increase. However, if there is private illiquid assets, \( y^i > 0 \), then \( x = \hat{x}_1 \) in (53) is greater than \( x_1 = x_2 = \hat{x}_1 \) in (49) as long as \( \theta < 1 \). Thus there is a benefit of holding excess reserves. In the Liquidity-trap equilibrium the rate of return in liquid assets must be the same as the rate of return in illiquid assets because excess reserves are required in equilibrium. Thus rates of return in liquid assets would increase while rates of return in illiquid assets would decrease as equilibrium allocation moves from Asset-shortage.

\[\text{However, if a floor system which directly sets interest rates for reserves is available, then nominal interest rates are achieved exactly by setting the interest on reserves as same as the nominal interest rate target. Thus there would exist no jump in nominal interest rates.}\]
Equilibrium to Liquidity-trap equilibrium. Thus the price of private assets would also increase and it would relax equilibrium condition (53).

4.3 Optimal Monetary Policy

When we add up expected utilities across agents in a stationary equilibrium, our welfare measure is

\[ W = \rho \{ u(x_1) - x_1 \} + (1 - \rho) \{ \theta u(x_2) - x_2 \} \]  

(56)

that represents the sum of the trade surpluses in the DM. Note that the first-best is \((x_1^*, x_2^*)\) where \(x_1^* > x_2^*\) and marginal utility of each type buyer is same when the allocations are along the curve with \(u'(x_1) = \theta u'(x_2)\).

In order to know whether the liquidity-trap equilibrium is optimal, I consider the optimal monetary policy in a neighborhood of the equilibrium allocation with \(z = \theta\), since the consumption level of liquidity-trap equilibrium would be greater as long as \(\theta < 1\). Since the welfare function and feasible allocations in the (49) can be described in a \((x_1, x_2)\) plain, I compare their slopes at \(x_1 = \tilde{x}_1\).

At \(x_1 = \tilde{x}_1\) the allocation satisfies with binding incentive constraint, \(x = x_1 = x_2\). Thus in the \((x_1, x_2)\) plain the slope of welfare function at \(x = x_1 = x_2\) is

\[ \frac{\partial x_2}{\partial x_1} = -\frac{\rho \{ u'(x) - 1 \}}{(1 - \rho) \{ \theta u'(x) - 1 \}} < -\frac{\rho}{(1 - \rho) \theta}. \]  

(57)

The slope of the government budget constraint (49) at \(x = x_1 = x_2\) is

\[ \frac{\partial x_2}{\partial x_1} = -\frac{\rho \{ u'(x) + xu''(x) \}}{(1 - \rho) \theta \{ u'(x) + xu''(x) \} - K'(x_2)} > -\frac{\rho}{(1 - \rho) \theta}. \]  

(58)

where \(K(x_2) = \frac{\beta y \theta u'(x_2)}{1 - \beta \theta u'(x_2)}\). Since \(u'(x) + xu''(x) > 0\) and \(K'(x_2) < 0\), the slope of welfare function is steeper at liquidity trap equilibrium. It implies that the optimal equilibrium allocation is achieved at the Liquidity-trap equilibrium.

5 Discussion

It is certain that the liquidity trap - a situation in which the implementation of monetary policy cannot influence in the market and real economy - is a serious concern for policy makers. In the history of the Great Depression the short-term interest rates decreased to zero in 1930-1932 and remained at zero for several years. Excess reserves in the banks increased and bank credit failed to
expand until 1936-1937. Brunner and Meltzer (1968) suggest as an alternative that a trap could have been operated within the banking system when banks desired to hold excess reserves and were unwilling to lend. It is also shown that the Federal Reserve bank officers had this situation in mind: Chairman Eccles testified at the U.S. Congress in 1935 that, even if currency was used to purchase government bonds from the public, there would be no increase in the money supply or in bank credit.

Mr. Cross: “Why not pay off all government bonds and get rid of paying any interest—because that would be inflation itself?”

Governor Eccles: “Here is what would happen... such-action would simply increase the reserves of the banking system by the amount of government bonds which were purchased with currency. The currency would go out, if it was $10 billion or $20 billion or $3 billion, whatever amount the government paid out in currency to retire its bonds; but the currency would immediately go into the banks and from the banks into the Federal Reserve banks... and you would have; additional reserves, additional excess reserves...”

It is hard to confirm that there was a liquidity trap in the Great Depression and that this liquidity trap occurred because banks desired to hold excess reserves. However, this paper shows that there is a possibility of a liquidity trap when banks have an incentive to hold liquid assets in their balance sheet.

6 Conclusion

In the paper I construct a banking model to study how private information confines liquidity insurance and the implementation of monetary policy. Given idiosyncratic liquidity shocks, lack of memory generates private information on types. A truth-telling banking contract is offered to provide liquidity efficiently under private information. When the supply of total assets is not enough to support liquidity distribution, the truth-telling incentive constraint binds and a liquidity premium arises in the price of illiquid assets. In the extended model with monetary policy when the truth-telling constraint binds, there exists a liquidity trap in which open market operations are ineffective in real allocations. This liquidity-trap equilibrium is different from the previous ones with currency-shortage or zero-lower-bound because it is generated by the incentive of banks to hold illiquid and even liquid assets for efficient liquidity provision.

This paper takes a step forward to understand the liquidity trap. It provides a model in which the liquidity trap can exist when banks have an incentive to hold liquid assets in their balance sheets so that it opens a possibility of studying the liquidity trap further. But it also leaves further questions unanswered. For example, liquid assets in a bank’s balance sheet can play a
role in preventing bank runs. In this respect we can ask how fragility of banks is associated with the effectiveness of monetary policy. This question requires a deeper consideration and explicit modeling on bank runs as a part of the model for the liquidity trap.

7 References


Figure 1. Timing

$CM_t$  
$DM_t$

receive transfer  
meet banker  
meet seller

pay debt  
deposit to banker  
type i known

market opens  
market closes


Figure 2. Perfect Information

\[
\begin{align*}
1 - \beta \rho x^* & \quad \frac{1}{\beta}
\end{align*}
\]
Figure 3. Single Crossing Property
Figure 4. Private Information

\[ z = \frac{\rho}{(1 - \rho)} \]

\[ y \]

\[ \frac{1 - \beta x^*}{\beta} \]

\[ \frac{1 - \beta px^*}{\beta} \]

\[ \frac{1 - \beta x^*}{\beta} \]
Figure 5. Liquidity Trap Equilibrium
Figure 6. Treasury Bill Rates and Excess Reserves in Great Depression

Treasury Bill Rates

Excess Reserves
Figure 7. Optimal Monetary Policy