Aggregate Risk, Inside Money, and Bank Capital Requirements *

(Job Market Paper)

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Abstract

This paper develops a new theory of bank capital requirements to provide insight into credit-cycle stabilization and macro-prudential policy. A general equilibrium banking model is constructed in which deposit claims backed by bank assets support secured credit arrangements with limited commitment. The competitive equilibrium allocation is constrained-suboptimal because the asset market is imperfect when assets are useful for exchange with limited commitment. This paper shows that given aggregate risk, pro-cyclical capital requirements can improve economic welfare by trading off the opportunity cost of holding additional bank capital for the benefit from sharing liquidity risk. In the model bank capital requirements can influence real interest rates on assets and the inflation rate by adjusting the pledgeability of assets. Thus capital requirements can be an effective policy tool when the conventional monetary policy is limited to exchange liquid and illiquid assets.

Key Words : contingent contract, consumption risk-sharing, procyclical bank capital requirement **JEL Codes** : E42, E52, E58

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1 Introduction

Why do we need to impose bank capital requirements in reality? A conventional rationale for bank capital requirements is based on deposit insurance: banks will tend to take too much risk under this safety net, so bank capital requirements are needed to correct this moral hazard problem created by deposit insurance. Alternatively, it is sometimes argued that bank capital requirements can be justified based on an externality associated with systemic risk. For example, contagion can justify government interventions since a default of one bank would lead to a chain reaction where many other financial intermediaries could go bankrupt. However, deposit insurance may not be a concrete argument for capital requirements because there could be a better solution: lender of last resort policies can prevent bank runs without raising moral hazard of banks to take an excessive risk. Also, as long as it is costly to avoid default, fire sales and contagion, the rationale for correcting externality cannot be taken as axiomatic because it is still required to analyze the cost and benefit of the regulation.

In this paper I develop a novel mechanism by which bank capital requirements can improve economic welfare by promoting efficient liquidity provision. In particular, with aggregate risk and limited commitment, bank capital requirements can play a role in sharing liquidity risk by adjusting the pledgeability of assets. This role of bank capital requirements can provide insight into credit-cycle stabilization and so-called macro-prudential policy. This framework is also suitable for welfare analysis in general equilibrium as the cost of holding bank capital is determined endogenously in the model without externalities.

In order to explore this issue I develop an asset-exchange model in which bank liabilities are used to facilitate payments and settlement in an explicit way. This micro-founded model has the advantage of incorporating informational frictions such as limited commitment and imperfect memory easily and is highly tractable, with an array of assets and a contingent form of banking contract. The basic structure of the model comes from Rocheteau and Wright (2005), in which ex ante heterogeneous agents can trade in the decentralized meetings and their asset portfolios are rebalanced in the centralized markets. The structure of banking arrangement is borrowed from Williamson (2012), where bank liabilities are protected only by the value of bank assets with limited commitment. There is a fixed supply of private assets for which the returns are subject to aggregate risk. Given the aggregate risk, a contingent banking contract is considered to maximize the ex ante expected value of depositors under perfect competition.

Limited commitment is a key element in the model, as it can restrict credit provision by banks.[1] Since assets are useful for supporting these credit arrangements, the price of the assets can be valued not only for their expected stream of future yields, but also for the usefulness in exchange. This

[1] Unlike government debt, which is supported by the power of forcing tax, bank liabilities are only protected by the collateralized assets under limited commitment of banks.

gives rise to a liquidity premium in the model.

Given the competitive asset market, the equilibrium allocation can be constrained when the limited commitment matters. Then the constrained competitive equilibrium allocation may not be constrained-optimal according to the result of Geanakoplos and Polemarchakis (1986).[2] It is because the asset market is imperfect when assets are useful for exchange with limited commitment, even though the contract is complete and there is no externality. Thus there is an open possibility of welfare improvement by manipulating the degree of limited commitment.

This paper makes three key contributions. First, there exists an equilibrium in which bank capital can be held voluntarily even though it is costly to hold. Since bank capital is not useful for exchange, it is costly for a bank to hold assets to support bank capital when asset prices reflect a liquidity premium. However, if the assets are plentiful only in the high-return state, the *ex post* marginal benefit of holding assets will be less than the *ex ante* marginal cost of buying assets. Then bank capital is useful in the view of depositors because they can avoid holding unnecessary assets in the high-return state. This result can provide an alternative explanation for the historical fact that banks have at times held capital in excess of capital requirements. Berger, Herring, and Szego (1995) report that in the 1840s U.S. commercial banks had equity-to-asset ratios of over 50 percent and this ratio declined over time, but it has been kept above the required level even after the Basel I capital requirement was imposed in the 1990s. This excessive holding of bank capital has brought about theories for another role of bank capital. For example, Diamond and Rajan (2000) present a model in which voluntarily held bank capital serves as a buffer in recessions to prevent bank runs. In this paper I show that strictly positive bank capital can exist in equilibrium without additional functions of bank capital.

Secondly, it is shown that, given aggregate risk, imposing a bank capital requirement in the high-return state can improve welfare when depositors are sufficiently risk-averse. Requiring additional bank capital seems to make things worse because it reduces the pledgeability of assets, i.e. only a proportion of assets serve as collateral, so that secured credit is constrained further. However, restricting the pledgeability of assets in the high-return state can affect *ex ante* asset prices because the liquidity premium on the assets, which is associated with trade inefficiency in each state, will change. Then the consumption level in the low-return state can increase since the limited commitment constraint is relaxed as the asset price rises. This mechanism implies that purchasing power available to depositors can shift from the high-return state to the low-return state without a real transfer. If the depositors are risk-averse enough, then bank capital requirements should be raised until the marginal cost of holding additional bank capital and the marginal benefit of sharing consumption risk are equal.

This main result has implications in terms of pro-cyclical bank capital requirements. The

[2] Geanakoplos and Polemarchakis (1986) show that the equilibrium allocation is constrained suboptimal in a model of competitive general equilibrium with incomplete markets.

previous justification for pro-cyclical requirements is based on systemic risk. For example, the counter-cyclical buffer in the Basel III accord, which requires additional bank capital in a period of excess credit growth, is proposed to reduce a social cost associated with default of banks in recessions. In this paper I find that the same pro-cyclical capital requirement is beneficial for society, but it is beneficial because this pro-cyclical requirement can stabilize credit cycles.

Finally, bank capital requirements can influence macroeconomic variables and the implementation of monetary policy. In order to study this issue I extend the model by introducing two additional government-issued assets, money and government bonds, and by borrowing a monetary policy framework from Williamson (2014), in which the real value of outstanding government debts is kept as constant and the central bank chooses the proportion between money and government bonds through open market operations. There is an idiosyncratic shock faced by depositors under which one type of depositor must use currency for trade and the other type can make a credit arrangement with government bonds and private assets. Since bank capital requirements can affect the liquidity premium on the asset prices, real interest rates on the assets can be adjusted without using open market operations. This path allows us to consider bank capital requirements as an unconventional monetary policy tool at the zero-lower-bound where conventional monetary policy is limited. Additionally, it is shown that, with monetary policy fixed, the feasible set of equilibrium allocations can be reduced by imposing bank capital requirements: if the liquidity premium on the backed assets rises with bank capital requirements then the inflation rate must rise in equilibrium to make rates of return on currency and government bonds equal. Thus given the same credit arrangements, the amount of currency trade can decrease by imposing bank capital requirements.

This paper is related to the literature that studies the necessity of bank capital regulations in a theoretical way.[3] One strand of the literature focuses on the moral hazard of banks induced by deposit insurance. For example, Kareken and Wallace (1978) show that deposit insurance can create moral hazard of banks. Kim and Santomero (1988) and Furlong and Keeley (1989) conduct a pioneering study on the optimal risk-taking problem of banks by using a mean-variance model and by considering the option value of deposit insurance, respectively. Dewatripont and Tirole (2012) and Boyd and Hakenes (2014) concentrate more on managerial looting incentive than risk-taking behavior. The other strand of the literature, for example Lorenzoni (2008) and Jeanne and Korinek (2011), emphasizes pecuniary externality: when banks cannot internalize externalities, the distributive conflict among agents can arise in the incomplete market. Thus an intervention is necessary to correct this pecuniary externality problem. In the present paper bank capital requirements are rationalized by the imperfection of asset markets with limited commitment instead of agency problem and pecuniary externality in incomplete markets.

Limited commitment is also emphasized in Gertler and Kiyotaki (2013) in which bank capital, i.e. net worth, is helpful to raise funds from depositors by overcoming limited commitment. How-

[3] See also VanHoose (2007) for a literature review on banking theories with the bank capital regulations.

ever, in this paper bank capital is important not because it has a different function, but because it is just not useful for exchange so that it can reduce the liquidity of assets. The function of bank liabilities as a liquidity provider is also shown in Begenau (2015) where bank capital requirements can in fact increase bank lending because the reduced supply of bank debt adjusts the interest rate downwards. In the paper bank capital is held only upon capital requirements because bank capital is useless for liquidity. But in this paper bank capital can be held voluntarily although it is costly to hold bank capital.

This paper builds on the literature that provides micro-foundations for monetary economics as pioneered by Kiyotaki and Wright (1989) and Lagos and Wright (2005). Banking models with explicit trade frictions are developed by Freeman (1988), Champ, Smith and Williamson (1996) and Sanches and Williamson (2010). The role of assets in exchange is studied by Geromichalos, Licari and Suarez-Lledo (2007), Lagos and Rocheteau (2008). Limited commitment in assets-exchange is studied by Kiyotaki and Moore (2005), Venkateswaren and Wright (2013). Aggregate risk in the return of assets is introduced in Lagos (2010) to explain the equity-premium puzzle and in Andolfatto, Berentsen and Waller (2014) to consider the optimal information disclosure. Bank capital is recognized as a non-pledgeable part of assets in Williamson (2014), but the rationale for bank capital requirements is considered in the present paper.

In the second section I describe the elements of the model. In the third section a simple model with one risky asset is characterized and analyzed with bank capital requirements. I introduce money and government bonds in the fourth section to consider the relationship between bank capital requirements and monetary policy. The final section concludes.

2 Model

The model structure is based on Rocheteau and Wright (2005). Time $t = 0, 1, 2, \ldots$ is discrete and the horizon is infinite. Each period is divided into two sub-periods - the centralized market (CM) followed by the decentralized market (DM). There is a continuum of buyers, sellers and bankers, each with unit mass. An individual buyer has preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t [-H_t + u(x_t)],$$

where $H_t \in \mathbb{R}$ is labor supply in the CM, $x_t \in \mathbb{R}_+$ is consumption in the DM, and $0 < \beta < 1$. Assume that $u(\cdot)$ is strictly increasing, strictly concave, and twice continuously differentiable with $u^{'}(0) = \infty, u^{'}(\infty) = 0$, and $-x\frac{u^{''}(x)}{u^{'}(x)} = \gamma < 1$.[4] Each seller has preferences

[4]Constant relative risk aversion is useful to derive the benefit of consumption risk-sharing explicitly in the model. It is also useful to have a unique equilibrium because the demand for assets is strictly increasing in rates of return so that substitution effects dominate income effects when $\gamma < 1$.

$$E_0\sum_{t=0}^{\infty}\beta^t[X_t - h_t],$$

where $X_t \in \mathbb{R}$ is consumption in the *CM*, and $h_t \in \mathbb{R}_+$ is labor supply in the *DM*. An individual banker has preferences

$$E_0\sum_{t=0}^{\infty}\beta^t[X_t - H_t],$$

where $H_t \in \mathbb{R}_+$ is labor supply in the *CM*, $X_t \in \mathbb{R}_+$ is consumption in the *CM*.

Buyers can produce in the *CM*, but not in the *DM* while sellers can produce in the *DM*, but not in the *CM*. Bankers can produce and consume in the *CM*, but cannot participate in the *DM*. One unit of labor input produces one unit of perishable consumption good either in the *CM* or in the *DM*.

In this economy there are two kinds of public assets, fiat money and one-period government bonds, issued by the fiscal authority. Fiat money trades at price ϕ_t in terms of goods in the *CM* of period t. One-period maturity government bonds, which are obligations to pay one unit of fiat money in the *CM* of period $t+1$, sell at price z_t in terms of goods in the *CM* of period t. There also exists one private asset - a divisible Lucas tree. It is endowed to buyers in the *CM* of the initial period $t = 0$ with a fixed unit supply. The Lucas tree pays off y_t units of consumption goods as a dividend and trades at the price ψ_t in terms of goods in the *CM* of period t. The dividend of the Lucas tree, y_t, is an $i.i.d$ random variable which can take on two possible values, $0 < y^l \leq y^h < \infty$. Let π denote the probability of a high dividend y^h, and let $\bar{y} \equiv \pi y^h + (1-\pi)y^l$ as an expected payoff of this random dividend.

In the beginning of the *CM*, all agents meet together and debts or obligations are paid off. Buyers receive lump-sum transfer(or pay lump-sum tax) and the holders of the Lucas tree receive the dividends. Then a Walrasian market open, goods are produced, assets are traded and buyers deposit goods or assets into a banker with a contingent deposit contract. In the end of the *CM*, the asset market is closed and the dividend of the Lucas tree in the next period $t+1$ is known in the end of the period t *CM*.

In the *DM* each buyer meets each seller bilaterally and the terms of trade are determined by bargaining. The buyer makes a take-it-or-leave-it offer to the seller. There is no record-keeping technology in the *DM* so that agents are anonymous. Limited commitment is assumed so that no one can be forced to work. Thus no unsecured credit is available, recognizable assets are essential for trade, and trade must be *quid pro quo*.

In a manner similar to Sanches and Williamson (2010), there are two kinds of random matches in the *DM*. In a fraction ρ of non-monitored *DM* meetings fiat money is only recognized by sellers. In $1-\rho$ fraction of monitored *DM* meetings the entire asset portfolio held by the buyer can be verified by the seller so that secured credit arrangement is available for trade. I assume that fiat money, i.e. currency, is portable and can be used on the spot in the *DM* while the other assets are not.[5] Thus deposit claims backed by the assets can be used on the spot to transfer account balances of the buyer to the seller in the monitored *DM* meetings. Since deposit claims issued by buyers or sellers can violate no record-keeping environment in the *DM*, I assume that a representative banker provides a banking arrangement by issuing deposit claims.[6] Note that perfect competition is assumed among the bankers so that a banker suggests a deposit contract that provides the maximum expected value of depositors.

Given no memory and limited commitment, the banker can run away in the next *CM*, but the backed assets are seized and transferred to the seller.[7] Thus the asset portfolio except for currency can be pledged as collateral as shown in Kiyotaki and Moore (2005) or Venkateswaran and Wright (2013). One difference from their models is that the pledgeability of the assets can be chosen by imposing contingent bank capital requirements. Thus when a representative banker offers a contingent deposit contract, in which the payoff of deposit claims can vary by states, a proportion of the assets which backs the deposit claims can be adjusted by imposing bank capital requirements.[8]

When the contract term is arranged buyers do not know what types of meeting they will be in the next *DM.* Thus the banking contract also provides liquidity insurance as shown in Diamond and Dybvig (1983). Assume that the size of shock ρ is exactly observable and type is public information so that I can set aside of bank runs issue. After type is realized, type 1 buyers who will move to ρ non-monitored meetings can withdraw currency from the banker when they meet the banker and type 2 buyers who will move to $1-\rho$ monitored meetings remain with deposit claims. To support banking arrangement I assume that the buyer can meet only one banker in the *CM* after their liquidity shock is realized.[9]

The timing is as follows. In the beginning of *CM* debts are paid off and all buyers provide labor and trade assets and write a contract with a banker in a Walrasian market. After liquidity shock is realized buyers learn their type and ρ buyers meet the banker to withdraw money. In the end of *CM* the dividend for the next period is known for everyone. In the *DM* buyers meet sellers randomly in the bilateral meeting and make take-it-or-leave-it offers. In the next *CM* $1-\rho$ sellers can receive *CM* goods by redeeming deposit claims to the banker or sell them to buyers.

[5]Even if buyers can use their asset holdings directly for the trade, there is no more benefit from the direct asset-trade because a banker provides the optimal arrangement for buyers with zero profit.

[6]Since bankers have a linear utility function as same as buyers and sellers in the *CM*, there is no more advantage for using deposit claims of a banker instead of deposit claims of the other agents.

[7]All agents are subject to the same degree of limited commitment.

[8]The contract term must be state-contingent because no one knows the aggregate state when the contract is written.

[9]Note that if *ex post* asset-trading among buyers is allowed then banking contract is unraveled and collapsed as shown in Jacklin (1987).

[Figure 1 here]

2.1 Government

In the model the consolidated government consists of the fiscal authority and central bank. The fiscal authority issues one-period nominal government bonds in the *CM* and pays interests in the next *CM* while the monetary authority issues fiat money and injects (or absorbs) fiat money in the markets by exchanging fiat money with government bonds, i.e. open market operations. As well, the fiscal authority can collect lump-sum tax from buyers (or provide transfer to buyers) in the *CM*.[10] In period $t = 0$ government bonds are issued and fiat money is injected with lump-sum transfer, τ_0, and in the following periods outstanding fiat money and government bonds are supported by tax or transfer over time. So the consolidated government budget constraint for $t = 0$ is

$$\phi_0(M_0 + z_0 B_0) = \tau_0,$$

and for $t = 1, 2, 3, \ldots$

$$\phi_t\{M_t - M_{t-1} + z_t B_t - B_{t-1}\} = \tau_t$$

where M_t and B_t denote the nominal quantities of outstanding fiat money and government bonds held in the private sector in time t, respectively, and τ_t denote the real value of the lump-sum transfer to each buyer in period t. Government, fiscal authority or central bank, can impose exogenous bank capital requirements to the bankers.

3 Competitive Equilibrium with Lucas tree

In the model a representative banker is assumed to provide a liquidity management service to depositors. Given the aggregate risk the asset holdings can be valuable when the supply of assets is insufficient, but costly when the supply of assets is abundant. A banking arrangement can manage this liquidity provision problem by using a contingent bank capital claim. By providing a proportion of assets to a banker or the other agents when the assets are abundant and nothing when the assets are scarce, the liquidity for depositors can be managed efficiently. In this section this contingent banking arrangement is considered to maximize the expected utility of depositors. The optimal banking arrangement can describe when bank capital is held voluntarily even though bank capital is costly to hold.

In the subsections I analyze in what circumstance bank capital requirements can improve welfare. Since bank capital requirements, which require additional bank capital holdings for bankers,

[10] Tax or transfer is available only for consumption goods.

restrict the amount of liquidity for depositors in the economy. Thus these capital requirements are not helpful for liquidity provision in general. However, given the aggregate risk, bank capital requirements can be beneficial for smoothing the amount of liquidity across states. When the ex ante asset price reflects the liquidity premium in both states, restricting the liquidity in one state can increase the liquidity in the other state since the asset price is changed by the adjusted liquidity premium in both states.

To focus on these two main ideas I assume that there is no government assets and no reason for liquidity insurance by $\rho = 0$ in this section. Under perfect competition bankers suggest a contingent contract to maximize buyers' ex ante expected value. Thus in equilibrium a banker solves the following problem in the *CM* of period t:

$$\underset{d_t, a_t, x_t^h, x_t^l}{Max} - d_t + \pi u(x_t^h) + (1-\pi) u(x_t^l) \tag{1}$$

subject to

$$d_t - \psi_t a_t + \pi\{\beta(\psi_{t+1} + y^h) a_t - x_t^h\} + (1-\pi)\{\beta(\psi + y^l) a_t - x_t^l\} \geq 0 \tag{2}$$

$$\beta(\psi_{t+1} + y^h) a_t - x_t^h \geq 0 \tag{3}$$

$$\beta(\psi_{t+1} + y^l) a_t - x_t^l \geq 0 \tag{4}$$

$$d_t, a_t, x_t^h, x_t^l \geq 0 \tag{5}$$

All quantities in (1)-(5) are expressed in units of the *CM* good in time t. The problem (1) subject to (2)-(5) states that a contingent banking contract (d_t, x_t^h, x_t^l) is chosen in equilibrium to maximize the expected utility of a representative buyer subject to the participation constraint for the banker (2) and the incentive constraints for the banker by states (3)-(4) and non-negativity constraints (5). In (1)-(5) d_t denotes the quantity of goods deposited by the buyer, a_t denotes the demand of the banker for asset holdings, and x_t^i represents the consumption level of the buyer in each state i for $i = h, l$. The quantity on the left side of (2) is the net payoff for bankers. In the *CM* of time t the banker receives d_t consumption goods, issues a deposit claim, and invests in the private asset with market prices, $\psi_t a_t$. In the following *CM* the banker pays x_t^h or x_t^l to the holders of the deposit claim by the state h or l. The incentive constraints (3)-(4) imply that when deposit claims are paid off, the net payoff for the banker is greater than zero, the value that the banker could earn when he or she decides to abscond.

Note that if the limited commitment constraints (3) or (4) does not bind then bank capital, i.e. asset portfolio minus deposit, is strictly positive in (2) because the ex ante profit for bankers must be zero under perfect competition. As well note that since a state-contingent contract is considered in the problem, the banker can also choose non-contingent contract as an optimal choice, if needed.

Government can impose contingent bank capital requirements (δ^h, δ^l) in which a banker must set aside at least $\delta^i \in [0, 1)$ proportion of the asset portfolio by the state i. Then we can have additional bank capital constraints by states,

$$\beta(\psi_{t+1} + y^h)(1 - \delta^h)a_t - x_t^h \geq 0 \tag{6}$$

$$\beta(\psi_{t+1} + y^l)(1 - \delta^l)a_t - x_t^l \geq 0 \tag{7}$$

where the deposit claim is only pledgeable by $1 - \delta^i$ proportion of the assets in the state i.

Note that for $\delta^i = 0$ the bank capital constraints (6)-(7) are simply same with the limited commitment constraints (3)- (4), respectively. For $\delta^i \in (0, 1)$ if the bank capital constraints (6)-(7) do not bind, the limited commitment constraints (3)-(4) always do not bind while if the bank capital constraints (6)-(7) bind then the limited commitment constraints (3)-(4) are relaxed, respectively. Thus given $\delta^i \in [0, 1)$ an equilibrium can be constructed only with the bank capital constraints (6)-(7) that replace the limited commitment constraints (3)-(4) without loss of generality. Notice that bank capital requirements, δ^h and δ^l, are choice variables of government, thus no bank capital requirements with $\delta^h = \delta^l = 0$ can also be chosen at the optimum.

The first step is to solve the problem (1) subject to (2),(5)-(7) to characterize equilibrium. The constraint (2) must bind, as the objective function is strictly increasing in both x_t^h and x_t^l while (2) is strictly decreasing in both x_t^h and x_t^l. Since I will concentrate on the cases either constraint (6) or (7) binds, let λ^h and λ^l denote the multiplier associated with the incentive constraints (6) and (7), respectively. Then by plugging (2) into (1) we have the first-order conditions by a_t, x_t^h, x_t^l,

$$\psi_t = \pi\beta(\psi_{t+1} + y^h)\{1 + \lambda^h(1 - \delta^h)\} + (1 - \pi)\beta(\psi_{t+1} + y^l)\{1 + \lambda^l(1 - \delta^l)\}, \tag{8}$$

$$\pi\{u^{'}(x_t^h) - 1\} = \lambda^h, \tag{9}$$

$$(1 - \pi)\{u^{'}(x_t^l) - 1\} = \lambda^l \tag{10}$$

which can be reduced into

$$\psi_t = \pi\beta(\psi_{t+1} + y^h)\{(1 - \delta^h)u^{'}(x_t^h) + \delta^h\} + (1 - \pi)\beta(\psi_{t+1} + y^l)\{(1 - \delta^l)u^{'}(x_t^l) + \delta^l\} \tag{11}$$

The first-order condition (11) states that the net payoff to the banker from acquiring one unit of the asset is zero in equilibrium. In equilibrium a representative bank holds all the assets in its portfolio so that the asset market clear in the CM with

$$a_t = 1 \tag{12}$$

for $t = 0, 1, 2, \ldots$. The market clearing condition (12) states that the supply of the private asset is equal to the banker's demand.

***Definition** 1: Given (π, y^h, y^l) and bank capital requirements (δ^h, δ^l), a stationary competitive equilibrium consists of quantities (x^h, x^l) and asset price ψ and multipliers (λ^h, λ^l) which satisfy equations (6)-(10), (12).*

Note that there are five variables to be determined in a stationary equilibrium in Definition 1 and five equations with the asset market clearing condition are provided. Thus equilibrium allocations are determined by given parameters and bank capital requirements. From now on I will eliminate t subscripts to confine the attention to stationary equilibrium allocations.

3.1 No Bank Capital Requirements

In this subsection I characterize the equilibrium allocations with no bank capital requirements, $\delta^h = \delta^l = 0$, as a benchmark. Then it will matter for the determination of equilibrium whether the incentive constraints (3)-(4) bind or not. Thus I will consider each of the three relevant equilibrium cases: Neither constraint binds; The constraint for state l only binds; Both constraints bind. Note that there is no equilibrium case in which the constraint for state h only binds since $y^h \geq y^l$ is assumed given $\delta^h = \delta^l = 0$.

3.1.1 Neither constraint binds

In this case, since $\lambda^h = \lambda^l = 0$, from (8)-(10) we have $\psi = \psi^f$ and $x^l = x^h = x^*$ in equilibrium where $\psi_f \equiv \frac{\beta \bar{y}}{1-\beta}$ and x^* satisfies with $u^{'}(x^*) = 1$. The quantity of bank deposits, d, is fixed as x^* in the participation constraint (2) since (2) holds with equality in equilibrium. The efficient allocation, i.e. the first-best allocation, is attained when both incentive constraints do not bind. That means, given limited commitment, if the supply of the asset is sufficient in an economy, the efficient allocation can be supported. Given $\delta^h = \delta^l = 0$ if the incentive constraint for state l (4) does not bind then the incentive constraint for state h (3) does not bind as well. Thus it is required to have

$$\beta(\psi^f + y^l) - x^* \geq 0 \tag{13}$$

to support the efficient allocation as equilibrium. Equation (13), which can be transformed into $\beta y^l + \beta^2 \pi (y^h - y^l) \geq (1-\beta) x^*$, implies that the efficient allocation is attainable in equilibrium as the expected payoff of the dividend is large enough given the aggregate risk, i.e. $y^h - y^l$. This equilibrium is described as region 1 in Figure 2.

Note that holding an asset is not costly in this case since the real return of the asset is same as the inverse value of time preference with $\frac{\psi^f+\bar{y}}{\psi^f}=\frac{1}{\beta}$. Thus the bank capital, the asset holdings minus bank deposits, is determined as $\psi^f - x^*$, but it is not costly to hold bank capital in this case.

3.1.2 The constraint for state l only binds

In this case since $\lambda^l > \lambda^h = 0$, (4) and (8) can be transformed into

$$\beta(\psi + y^l) - x^l = 0 \tag{14}$$

and

$$\psi = \pi\beta(\psi + y^h) + (1-\pi)\beta(\psi + y^l)u^{'}(x^l), \tag{15}$$

respectively. Then the incentive constraint (14) and the first-order condition (15) solve for ψ and x^l in equilibrium. Since the incentive constraint for state l binds, the consumption level in state l is lower than the optimal level, $x^l < x^*$ and a liquidity premium in the asset price arises so that the asset price is greater than its fundamental value, $\psi > \psi_f$, in equilibrium with $u^{'}(x^l) > 1$. Since the incentive constraint for the state h (3) does not bind we have $x^h = x^*$ in equilibrium.[11] The quantity of bank deposits, d, is determined as $\psi - \pi\{\beta(\psi + y^h) - x^*\}$ in the participation (2) while the bank capital is $\pi\{\beta(\psi + y^h) - x^*\}$ which is at least positive. Note that both bank deposits and bank capital increase in ψ. This is because when the incentive constraint binds, the asset price rises so that the balance sheet of the banker expands. Additionally, note that even though assets are plentiful in the state h, the asset price, ψ, is greater than its fundamental value, ψ^f, because the asset price, which is determined before the state is realized, also reflects the liquidity premium in the state l.

In this case since there exists a liquidity premium in the asset price, the real return of the asset is lower than the inverse value of time preference with $\frac{\psi+\bar{y}}{\psi} < \frac{1}{\beta}$. This implies that holding the asset is costly and so is holding bank capital. However, the bank capital is voluntarily held by the banker in equilibrium since the marginal benefit of holding assets in the state h ex post is lower than the marginal cost of acquiring the asset ex ante. When the state h is realized the marginal benefit of holding extra assets, i.e. the total value of asset portfolio minus the asset used for trade - $\beta(\psi + y^h) - x^*$, is lower than one because the marginal utility of consumption with those extra assets is lower than one with $u^{'}(x^*) = 1$. But the marginal cost of acquiring total asset portfolio is one because the marginal utility of labor supply or consumption good in the CM is fixed as one in this quasi-linear model. In order to maximize depositor's expected value the banker will not let depositors hold these extra assets in the state h ex post. Since the profit of the banker is always

[11] This case can be generalized with a continuous distribution for dividends. If the variance of dividend distribution is large enough then we will have a measure of h state in which the incentive constraint does not bind.

zero in equilibrium, it is optimal for the banker to hold bank capital for depositors even though it is costly.[12] As a consequence bank capital, which is costly to hold, is determined as strictly positive in equilibrium. It implies that bank capital, which is not useful for trade, needs to be held for efficient liquidity management when there exists an aggregate risk in assets and the limited commitment constraint binds.

For this to be an equilibrium, ψ and x^l must satisfy with

$$\beta(\psi + y^h) - x^* \geq 0. \tag{16}$$

This implies that given the aggregate risk when the expected payoff of the dividend is small, but the incentive constraint for the state h does not bind, this equilibrium case is feasible and it is described as region 2 in Figure 2.

3.1.3 Both constraints bind

In this case, since $\lambda^l > 0$, $\lambda^h > 0$, the incentive constraint for the state h (3) and the first-order condition (8) can be transformed into

$$\beta(\psi + y^h) - x^h = 0 \tag{17}$$

and

$$\psi = \pi\beta(\psi + y^h)u^{'}(x^h) + (1-\pi)\beta(\psi + y^l)u^{'}(x^l), \tag{18}$$

respectively. Then the incentive constraints (14) and (17), and the first-order condition (18) solve for ψ, x^h, and x^l in equilibrium. Since both incentive constraints bind, the consumption level in the state l is lower than that in the state h, $x^l < x^h$, as long as $y^l < y^h$ holds, and a liquidity premium in the asset price arises so that the asset price is greater than its fundamental value, $\psi > \psi_f$. The quantity of bank deposits, d, is determined as ψ in the participation constraint (2) while the bank capital is zero because both incentive constraint bind. The bank capital would not be held by the banker because even in the state h the supply of assets is too scarce so that the marginal benefit of holding the asset is greater than one with $u^{'}(x^h) > 1$. Note that bank deposits increases in ψ as well, but bank capital is fixed as zero in this case because the dividends are too small. When the expected payoff of the dividend is too low given the aggregate risk, this equilibrium case is attainable and it is described as region 3 in Figure 2.

In the Figure 2 region 1 and region 2 are separated by a straight line, i.e. equation (13) with equality. The curve between region 2 and 3 is drawn on the points where $x^h = x^*$ just holds with

[12] Even though the bankers are risk-averse this logic can be applied similarly. The banker will hold extra assets as a bank capital in equilibrium as long as the marginal benefit of holding assets in the state h ex post is same as the marginal cost of holding assets ex ante.

zero bank capital in equilibrium. Thus the incentive constraint for the state h (16) holds with equality on this curve. Note that at $y^h = y^l$ region 2 vanishes since the two incentive constraints collapse into one constraint. Thus if there is no aggregate risk then there is no reason to hold costly bank capital for the banker in equilibrium. The dotted line in region 2 indicates the points that provide the same expected payoff of dividends with $\bar{y} = \pi y^h + (1-\pi)y^l$. This line is located below the borderline between region 1 and 2, because given the same level of $\bar{y}$, the incentive constraint for the state h (4) is constrained by the lower value of y^l as the aggregate risk, i.e. $y^h - y^l$, increases. Given the same expected payoff of dividend, when the aggregate risk increases the equilibrium allocation moves parallel to the dotted line.

[Figure 2 here]

3.2 Bank Capital Requirements

In this subsection I consider in what circumstance bank capital requirements can improve welfare. Given the contingent bank capital requirements, we can set either $\delta^h > 0$ or $\delta^l > 0$. If symmetric bank capital requirements with $\delta = \delta^h = \delta^l > 0$ are enforced then the welfare of the equilibrium allocation becomes worse. It is because the symmetric bank capital requirements have the same effect with reducing the supply of assets from one to $1-\delta$. Thus the consumption levels in both states strictly decrease when the symmetric bank capital requirements are implemented. Then we can consider two asymmetric capital requirements, either $\delta^h > \delta^l = 0$ or $\delta^l > \delta^h = 0$.[13] In the following I focus on the bank capital requirements with $\delta^h > \delta^l = 0$ to verify whether these requirements can be beneficial or not, and show that the bank capital requirements with $\delta^l > \delta^h = 0$ cannot improve welfare. From now on I assume that $\delta^l = 0$ and replace δ^h with δ. Since the equilibrium allocation is already efficient in region 1, I confine our attention to the region 2 and 3.

3.2.1 No Aggregate Risk

Let me begin with a special case, in which there is no aggregate risk with $y^h = y^l = \bar{y}$, in order to know the benefit of bank capital requirements. Since the aggregate risk is diversified the consumption levels in both states are equal as $x^l = x^h \equiv x$ in equilibrium. Then the first-order condition (11) can be transformed into

$$\psi = \beta(\psi + \bar{y})u^{'}(x), \tag{19}$$

and the incentive constraints (3) and (4) collapse to one incentive constraint. This constraint can be written as

[13] In case of $\delta^h > \delta^l > 0$ or $\delta^l > \delta^h > 0$ we can improve the welfare by subtracting δ^l or δ^h in both capital requirements, respectively.

$$\beta(\psi + \bar{y})(1 - \delta) - x \geq 0 \tag{20}$$

with asset market clearing condition, $a = 1$, in equilibrium. Since we are not interested in the equilibrium case of region 1, suppose that the bank capital constraint (20) binds with $\delta = 0$. The bank capital constraint (20) states that if $\delta > 0$ then the deposit claim is backed only by $1 - \delta$ proportion of the assets. Given δ, the first-order condition (19) and the incentive constraint (20) with equality solves for ψ and x in equilibrium. Note that the equilibrium allocation is uniquely determined because ψ is strictly decreasing in x in (19) and strictly increasing in x in (20).

Lemma *1. If there is no aggregate risk and the incentive constraint binds, the welfare is strictly decreasing in δ.*

Proof. *If the incentive constraint (20) binds then $x > 0$ solves for $x(1 - \beta u'(x)) = (1 - \delta)\beta\bar{y}$ in equilibrium. Since $1 - \beta u'(x)$ is strictly increasing in x, x is strictly decreasing in δ* ■

Lemma 1 states that given limited commitment, if there is no aggregate risk then bank capital requirements cannot be beneficial. If bank capital requirements are effective in equilibrium, the banker needs to hold more capital than he/she would choose. Thus as long as holding assets is costly, bank capital requirements have a negative effect on the welfare by reducing the proportion of the assets which is useful for trade. Moreover, there is no positive effect of bank capital requirements on the welfare in this case. Note that in this case bank capital requirements are not contingent; It always restricts a fixed δ proportion of the asset. In this respect the reason that bank capital requirements cannot beneficial in this case can also be explained by the case of the symmetric bank capital requirements.

3.2.2 Aggregate Risk

Now consider a general case in which there exists an aggregate risk with $y^h > y^l$. Note that equilibrium in region 2 and equilibrium in region 3 are almost same except for that there exists a strictly positive bank capital in region 2. So I analyze mainly whether the welfare of the equilibrium in region 2 can be improved by bank capital requirements and show that the same argument can be applied for the equilibrium in region 3.

Suppose that an equilibrium in region 2 exists with a strictly positive bank capital given $\delta = 0$. Then there exists a threshold $\tilde{\delta} > 0$ at which the bank capital constraint (6) starts to bind; At $\delta = \tilde{\delta}$ we still have $\psi = \psi_f$ and $x^h = x^*$ in equilibrium and (6) holds with equality. Thus $\tilde{\delta}$ requires to satisfy with

$$\beta(\psi + y^h)(1 - \tilde{\delta}) - x^* = 0 \tag{21}$$

where ψ and x^l are the solution of the incentive constraint (14) and the first-order condition (15). By construction, for $0 \leq \delta \leq \tilde{\delta}$ bank capital requirements are not effective in real allocations because

(3) does not bind. Thus the equilibrium allocation is same as one with $\delta = 0$ and only bank capital is decreasing in δ. As a result bank capital requirements are not beneficial for $0 \leq \delta \leq \tilde{\delta}$ in region 2.

For $\delta > \tilde{\delta}$ bank capital requirements are effective in real allocations since the bank capital constraint (6) binds. In this case the first-order condition (11) can be written as

$$\psi = \pi\beta(\psi + y^h)\{(1-\delta)u^{'}(x^h) + \delta\} + (1-\pi)\beta(\psi + y^l)u^{'}(x^l) \tag{22}$$

which can be rearranged to

$$\pi x^h\{u^{'}(x^h) + \frac{\delta}{1-\delta}\} + (1-\pi)x^l u^{'}(x^l) = \frac{\pi\beta y^h\{(1-\delta)u^{'}(x^h) + \delta\} + (1-\pi)\beta y^l u^{'}(x^l)}{1 - \beta\pi\{(1-\delta)u^{'}(x^h) + \delta\} - (1-\pi)\beta u^{'}(x^l)} \tag{23}$$

Note that the left-hand side of (23) is strictly increasing in x^h and in x^l because $-x\frac{u^{''}(x)}{u^{'}(x)} = \gamma < 1$ while the left-hand side of (23) is strictly decreasing in x^h and in x^l, while . Thus we can rewrite (23) in the form of

$$F(x^h, x^l, \delta) = 0, \tag{24}$$

where the function $F(\cdot\,,\cdot)$ is strictly increasing in both arguments x^h, x^l given δ. Then the first-order condition (23) can be describe as the FOC curve in Figure 3.

Meanwhile, the two binding constraints (6) and (4) can be written with equality,

$$\beta(\psi + y^h)(1-\delta) - x^h = 0 \tag{25}$$

and

$$\beta(\psi + y^l) - x^l = 0, \tag{26}$$

respectively. Note that we have $x^h < x^*$ in equilibrium. Binding incentive constraints (25) and (26) can be reduced to

$$\beta(y^h - y^l) = \frac{x^h}{1-\delta} - x^l \tag{27}$$

where (27) can be described as the IC curve in Figure 3.

Note that the function $F(\cdot\,,\cdot)$ is strictly increasing in δ given x^h and x^l, because the left-hand side is strictly decreasing in δ whereas the right-hand side is strictly increasing in δ given x^h and x^l. Thus when δ increases the FOC curve shifts towards the origin from FOC_1 to FOC_2 as shown in Figure 3. On the other hand, as δ increases the IC curve (27) rotates counter-clockwise from IC_1 to IC_2 as shown in Figure 3. By rotating the IC curve x^h and x^l moves towards the 45 degree line where the consumption levels in both states are equal. Thus the bank capital requirement in the state h can show the consumption risk can be shared across the states.[14]

Let me briefly dicuss that the bank capital requirements with $\delta^l > \delta^h = 0$ cannot be beneficial. Suppose that the same equilibrium in region 2 exists with a strictly positive bank capital given $\delta^l = \delta^h = 0$ as above. If the bank capital requirements with $\delta^l > \delta^h = 0$ are implemented, then for $\delta^l > 0$ bank capital requirements are effective immediately. But since the incentive constraint for state h (16) remains relaxed with $x^h = x^*$ there is no benefit for consumption risk-sharing. It can be also confirmed by the following equilibrium conditions, in which the first-order condition and the incentive constraint can be transformed the bank capital requirements with $\delta^l > \delta^h = 0$ into,

$$\pi x^h u'(x^h) + (1-\pi)x^l\{u'(x^l) + \frac{\delta^l}{1-\delta^l}\} = \frac{\pi\beta y^h u'(x^h) + (1-\pi)\beta y^l\{(1-\delta^l)u'(x^l) + \delta^l\}}{1 - \pi\beta y^h u'(x^h) - (1-\pi)\beta y^l\{(1-\delta^l)u'(x^l) + \delta^l\}} \quad (28)$$

and

$$\beta(y^h - y^l) = x^h - \frac{x^l}{1-\delta^l}, \quad (29)$$

respectively. Note that when δ^l increases the first-order condition (28) moves into the origin as well, but the incentive constraint (29) rotates clockwise. Thus x^h and x^l shifts away from the 45 degree line so that it is worse even in a view of consumption risk-sharing. Hence the bank capital requirements with $\delta^l > \delta^h = 0$ cannot be beneficial.

[Figure 3 here]

Now I return to our main subject that the bank capital requirement in the state h with $\delta^h \equiv \delta > \tilde{\delta}$ can be beneficial when the benefit of consumption risk-sharing is greater than the cost of holding enforced bank capital.

Lemma *2. There exists a unique $\hat{\delta} > 0$ where $x^h = x^l$ at $\delta = \hat{\delta}$ in equilibrium.*

Proof. *As $\delta \to 1$, the equilibrium allocation approaches to $(0, \bar{x}^l)$ where $\bar{x}^l = min(x^*, x^l)$ such that x^l satisfies with the first-order condition (23) and the equilibrium condition (27) given $x^h = 0$ and $\delta = 1$ as shown in Figure 3. Note that the solution of this constrained maximization problem is continuous in δ since u and u' is continuous. Thus as shown in Figure 3, by the Intermediate Value theorem, there exists a point that $x^h = x^l$ holds at $\delta = \hat{\delta} \in (\tilde{\delta}, 1)$ in equilibrium. This point is unique because when δ increases x^h strictly decreases*■

Lemma 2 is helpful to earn Proposition 1. It states that as δ approaches to 1, x^h and x^l must pass the 45 degree line in which the consumption risk is perfectly shared. Thus we can just compare

[14]Note that when δ increases, x^h decreases in equilibrium, but we need to confirm that x^l can increase and the welfare can improve.

the point A with the point B in the Figure 3, because the welfare of the equilibrium located in the upper side of the 45 degree line is lower than the welfare of the point B. Note that the contract at $\delta = \hat{\delta}$ is a non-contingent debt contract because $x^h = x^l$ holds in equilibrium.

Proposition *1. In region 2 the optimal bank capital requirement δ^* exists in $(\tilde{\delta}, \hat{\delta}]$ when agents are sufficiently risk-averse with $\gamma > \gamma^*$.*

Proof. *Given $\delta = \hat{\delta}$, $\hat{x} \equiv x^h = x^l$ holds in equilibrium by construction. Then from the first-order condition (22) and the binding constraint (26), in equilibrium $\hat{x}$ must satisfies with*

$$\frac{1}{\beta}\hat{x} - y^l = \hat{\psi} = \hat{x}u^{'}(\hat{x}) + \pi\frac{\hat{\delta}}{1-\hat{\delta}}\hat{x} \tag{30}$$

where $\hat{\psi}$ denote the asset price in the equilibrium with $\delta = \hat{\delta}$. Since $\beta(y^h - y^l) = \frac{\hat{\delta}}{1-\hat{\delta}}\hat{x}$ holds from the equilibrium condition (27), the equation (30) can be transformed into

$$\frac{1}{\beta}\hat{x} - \hat{y} = \hat{x}u^{'}(\hat{x}) = (1-\gamma)u(\hat{x}) \tag{31}$$

where $\hat{y} \equiv \pi\beta y^h + (1-\pi\beta)y^l$. The second equality in (31) is derived by $-x\frac{u^{''}(x)}{u^{'}(x)} = \gamma$. Thus given $\hat{y}$ and γ, $\hat{x}$ is pinned down from (31). Given $\delta = \tilde{\delta}$, since the bank capital constraint (6) does not bind, the equilibrium conditions (14)-(15) and (21) can be transformed into

$$\tilde{\psi} = \pi x^* u^{'}(x^*) + \pi x^*\frac{\tilde{\delta}}{1-\tilde{\delta}} + (1-\pi)x^l u^{'}(x^l) = (1-\gamma)\{\pi u(x^*) + (1-\pi)u(x^l)\} + \pi x^*\frac{\tilde{\delta}}{1-\tilde{\delta}} \tag{32}$$

where $\tilde{\psi}$ denote the asset price in the equilibrium with $\delta = \tilde{\delta}$, similarly. Let's define $\bar{x} \equiv \pi x^* + (1-\pi)x^l$ and define $\tilde{x}$ as a certainty equivalent value between x^* and x^l by $u(\tilde{x}) \equiv \pi u(x^*) + (1-\pi)u(x^l)$. Then there exists a proportion $p \in (0,\pi)$ which satisfies with $\tilde{x} = \frac{p}{1-\tilde{\delta}}x^* + (1-p)x^l$ since u is strictly concave. Then (32) can be rewritten as

$$\frac{1}{\beta}\tilde{x} - \tilde{y} = \tilde{\psi} = (1-\gamma)\{\frac{\pi}{1-\tilde{\delta}}u(x^*) + (1-\pi)u(x^l)\} = (1-\gamma)u(\tilde{x}) \tag{33}$$

where $\tilde{y} \equiv py^h + (1-p)y^l$. Since p is strictly decreasing in γ , $\tilde{x}$ decreases from $\bar{x}$ to x^l as γ increases in $(0,\infty)$. Thus there exists a threshold γ^* where $\tilde{x} = \hat{x}$ holds. Given (π, y^h, y^l), if $\gamma > \gamma^*$ then $\tilde{x} < \hat{x}$. Finally, the welfare function in the equilibrium with $\delta = \hat{\delta}$ is $\hat{W} = u(\hat{x}) - \hat{x} + \bar{y}$ whereas the welfare function in the equilibrium with $\delta = \tilde{\delta}$ is expressed as $\tilde{W} = \pi\{u(x^*) - x^*\} + (1-\pi)\{u(x^l) - x^l\} + \bar{y}$. Since $\pi\{u(x^*) - x^*\} + (1-\pi)\{u(x^l) - x^l\} \leq \frac{\pi}{1-\tilde{\delta}}\{u(x^*) - x^*\} + (1-\pi)\{u(x^l) - x^l\} = u(\tilde{x}) - \bar{x} \leq u(\tilde{x}) - \tilde{x} < u(\hat{x}) - \hat{x}$ holds, $\hat{W} > \tilde{W}$ when $\gamma > \gamma^*$. Thus the optimal bank capital requirement δ^* exists in $(\tilde{\delta}, \hat{\delta}]$ because the welfare of the equilibrium allocation is continuous in δ since u is continuous and the solution of the problem is also continuous ■

Corollary *1. In region 3 the optimal bank capital requirement* δ^* *exists in* $(0, \hat{\delta}]$ *when agents are sufficiently risk-averse with* $\gamma > \gamma^*$.

Proof *When both incentive constraints (6) and (4) bind, for any* $\delta > 0$ x^h *is strictly decreasing in* δ *because (6) already binds. Thus the same proof for Proposition 1 can be applied with* $\tilde{\delta} = 0$ ■

Corollary 1 states that the same argument can apply for region 3 where the incentive constraint for state h already binds. It is because the consumption risk is also not shared when both constraints bind.

Proposition 1 provides a sufficient condition for beneficial bank capital requirements. Given the aggregate risk and the scarcity of assets, if the risk-aversion of depositors is greater than a threshold , $\gamma > \gamma^*$, no bank capital requirements are no more the optimal choice of the government. This result implies that bank capital requirements should be considered as a policy tool for consumption risk-sharing. Moreover, this result offers a justification of imposing a procyclical capital requirement, i.e., a counter-cyclical capital buffer in Basel III.[15] Note that this rationale is different from the reason that the regulators are motivated. The regulators suggest a counter-cyclical capital buffer to mitigate credit crunch by providing buffer in recession and to internalize the banks' incentive for the social cost of default in booms. However, in this model the procyclical capital requirement can also stabilize business cycle by reducing the amplitude of the variations in consumption.

The intuition behind Proposition 1 is as follows: Given the aggregate risk, the supply of assets varies across the states. Given the asset prices a banker can manage the cost of holding assets with bank capital, but the consumption risk is not shared since the real transfer across the states is hardly achieved. Bank capital requirements can be effective to smooth consumptions by tightening the constraint in the state h which can relax the constraint in the state l. However, this regulation takes a cost for holding an additional bank capital and it is more costly to hold bank capital as assets are scarcer since the liquidity premium rises further. Thus there are three factors which provide a sufficient condition for beneficial bank capital requirements. One is the risk aversion of depositors. As much as depositors are risk-averse, they will prefer to pay more for sharing the consumption risk. Secondly, the incentive for risk-sharing becomes stronger as the aggregate risk becomes larger. Finally, the level of asset scarcity is also important because if assets are too scarce, then bank capital becomes too costly to hold for risk-sharing.

Note that the market failure in the problem, which necessitates bank capital reqiurements, is not related to the deposit contract since a complete contingent contract is considered in the model. It is also not associated with an externality because a representative banker provides ex ante maximized contract to depositors under perfect competition, so the incentive of the banker are well aligned with the objective of the society. The market imperfection is caused by limited

[15] Basel Committee on Banking Supervision (2010a,b) introduce a new countercyclical component which varies from 0 percent to 2.5 percent at regulators' discretion in addition to the minimum total capital requirement.

commitment. Since the transactions in the decentralized market can be supported by the value of the collateral, if there arises a cost for holding the collateral by the scarcity of assets, the first welfare theorem does not apply any more.

The proposition 1 can be confirmed by a numerical example in Figure 4. Given parameter values the regions, with which the proposition 1 is satisfied, are indicated in Figure 4. The region does not include the cases in which assets are too scarce and it widens as the depositors become more risk-averse.

[Figure 4 here]

In the Figure 5 I show numerical examples of different equilibrium allocations: the equilibrium allocation in the first row panel graphs is generated for a benchmark in which the optimal bank capital requirement is zero. In this case when δ increases the liquidity risk is shared as x^h falls, x^l rises, but the welfare strictly decreases since the cost of holding bank capital is greater than the benefit of sharing risk. In the second row panel graphs the equilibrium allocation is changed as the total supply of assets increases. In this case the welfare can improve as δ increases because the cost of holding bank capital is lowered since the assets are less scarce. Similarly, the welfare can improve by imposing capital requirements when the buyers become more risk-averse as shown in the third row panel graphs and when the aggregate risk becomes greater as shown in the last row panel graphs. Thus these numerical examples confirm that bank capital requirements can be beneficial when assets are not too scarce and the depositors are sufficiently risk-averse and finally aggregate risk is large enough.

[Figure 5 here]

4 Monetary Equilibrium

In this section I introduce money and government bonds in the model to consider how bank capital requirements can influence the real macroeconomic variables and how they are associated with the implementation of monetary policy. In the previous section it was shown that bank capital requirements can have an impact on the asset price by adjusting the liquidity premium by states. Thus there is a possibility that given monetary policy fixed, bank capital requirements affect real interest rates on assets and the inflation rate. Since conventional monetary policy is limited at the zero-lower-bound, but if the real allocation can be changed by imposing bank capital requirements then we can influence macroeconomic variables with bank capital requirements even at the zero-lower-bound. This extension also shows what level of nominal interest rates imply that bank capital requirements will be beneficial. Since the currency trades in the non-monitored meetings are now activated, we should consider the effect of bank capital requirements not only on credit arrangements, but also on currency trades.

I assume that the dividend on assets is known after a buyer meets the banker to withdraw currency. This assumption allows us to characterize equilibrium in a simple way because the consumption of the buyer using currency does not depend on the state.

A representative banker solves the following problem in the CM of period t:

$$\underset{d_t,m_t,b_t,a_t,x_{1t},x^h_{2t},x^l_{2t}}{Max} - d_t + \rho u(x_1) + (1-\rho)\{\pi u(x^h_2) + (1-\pi)u(x^l_2)\} \tag{34}$$

subject to participation constraint

$$\begin{aligned} d_t - m_t - z_t b_t - \psi_t a_t + \{\frac{\beta\phi_{t+1}}{\phi_t}m_t - \rho x_{1t}\}+ \\ \pi\{\frac{\beta\phi_{t+1}}{\phi_t}b_t + \beta(\psi_{t+1}+y^h)a_t - (1-\rho)x^h_{2t}\}+ \\ (1-\pi)\{\frac{\beta\phi_{t+1}}{\phi_t}b_t + \beta(\psi_{t+1}+y^l)a_t - (1-\rho)x^l_{2t}\} \geq 0 \end{aligned} \tag{35}$$

and the limited commitment constraint for currency

$$\frac{\beta\phi_{t+1}}{\phi_t}m_t - \rho x_{1t} \geq 0 \tag{36}$$

and the limited commitment constraints for deposit claims by states

$$\frac{\beta\phi_{t+1}}{\phi_t}b_t + \beta(\psi_{t+1}+y^h)a_t - (1-\rho)x^h_{2t} \geq 0 \tag{37}$$

$$\frac{\beta\phi_{t+1}}{\phi_t}b_t + \beta(\psi_{t+1}+y^l)a_t - (1-\rho)x^l_{2t} \geq 0 \tag{38}$$

and the bank capital constraints by states

$$\{\frac{\beta\phi_{t+1}}{\phi_t}b_t + \beta(\psi_{t+1}+y^h)a_t\}(1-\delta^h) - (1-\rho)x^h_{2t} \geq 0 \tag{39}$$

$$\{\frac{\beta\phi_{t+1}}{\phi_t}b_t + \beta(\psi_{t+1}+y^l)a_t\}(1-\delta^l) - (1-\rho)x^l_{2t} \geq 0 \tag{40}$$

and non-negative constraints

$$d_t, m_t, b_t, a_t, x_{1t}, x^h_{2t}, x^l_{2t} \geq 0 \tag{41}$$

All quantities in (34)-(41) are expressed in units of the CM good in period t. The problem states that a contingent banking contract is chosen in equilibrium to maximize the expected utility of a buyer (34) subject to constraints (35)-(41). In (34)-(41), m_t and b_t denote the quantities of money and government bonds in terms of the CM good in period t held by the banker and x^i_{jt} denote the consumption of type j buyers in the state i in time t CM for $j \in \{1,2\}$ and $i \in \{h,l\}$. Unlike the secured credit arrangement, currency transactions of type 1 buyers are fully backed by real money balances with no risk in (36). Note that given the liquidity shock $\rho \in (0,1)$ this banking contract provides not only liquidity provision service by using deposit claims, but also liquidity insurance for each type.

From now on I focus on a stationary equilibrium where $\frac{\phi_{t+1}}{\phi_t} = \frac{1}{\mu}$ holds for all t, and μ denote the gross inflation rate. Moreover, we will restrict our attention to the cases in which the first-best allocation is infeasible. Then the participation constraint (35) and the incentive constraint for currency trade (36) and the incentive constraint for the state l (38) always bind, while the incentive constraint for the state h (37) may bind or not. In order to know whether the same argument for consumption risk-sharing can also be applied in this extended model I focus on the bank capital requirement in the high-return state, δ^h. So from now on let δ replace δ^h as we did in the previous section. As discussed in the previous section given $\delta \in [0,1)$ equilibrium can be constructed with the bank capital constraint (39) instead of the limited commitment constraint (37).

Without loss of generality, the first-order conditions by m, b, a can be attained as

$$\frac{\mu}{\beta} = u'(x_1) \tag{42}$$

$$z\frac{\mu}{\beta} = \pi\{(1-\delta)u'(x_2^h) + \delta\} + (1-\pi)u'(x_2^l) \tag{43}$$

$$\psi = \beta(\psi + y^h)\pi\{(1-\delta)u'(x_2^h) + \delta\} + \beta(\psi + y^l)(1-\pi)u'(x_2^l) \tag{44}$$

Incentive constraints (36),(38)-(39) can be rewritten by dropping t subscripts as

$$\frac{\beta}{\mu}m = \rho x_1 \tag{45}$$

$$\{\frac{\beta}{\mu}b + \beta(\psi + y^h)a\}(1-\delta) \geq (1-\rho)x_2^h \tag{46}$$

$$\frac{\beta}{\mu}b + \beta(\psi + y^l)a = (1-\rho)x_2^l \tag{47}$$

Note that if the bank capital constraint (46) does not bind, bank capital is strictly positive in equilibrium. In equilibrium asset markets clear in the CM for all t, so that the demands of the representative banker for currency, government bonds, and private assets are equal to the supplies of outstanding government assets and the fixed unit supply of Lucas tree, respectively, as

$$m_t = \phi_t M_t, \tag{48}$$

$$b_t = \phi_t B_t, \tag{49}$$

$$a_t = 1 \tag{50}$$

I assume that the fiscal authority keeps the total value of the outstanding consolidated government debt, V, constant forever. This requires a transfer $\tau_0 = V$ at $t = 0$. Then from the consolidated budget constraints we obtain the real term of lump-sum transfer,

$$\tau_t = \underbrace{(1 - \frac{1}{\mu})V}_{seigniorage} + \underbrace{\frac{1}{\mu}(z-1)b}_{real\,interest\,payment}, \quad t = 1, 2, 3, \ldots, \tag{51}$$

where τ_t is required to maintain the constant value of V for the consolidated government debt in every period. Note that the lump-sum transfer consists of seigniorage from inflation and real interest payment for government bonds. This fixed real value of consolidated government debt assumption allows us to separate monetary policy, specifically open market operations, from fiscal policy. Moreover, by assuming V as being small, we can explore the cases in which the first-best allocation is infeasible.

Since the policy rule of the fiscal authority for $t = 1, 2, 3, \ldots$ is fixed, all we need to consider for constructing equilibrium is the government budget constraint for $t = 0$ with $\tau_0 = V$,

$$m + zb = V \tag{52}$$

Definition *2: Given (π, y^h, y^l, ρ, V) and the nominal interest rate $\frac{1}{z} - 1$, bank capital requirements (δ^h, δ^l), a stationary monetary equilibrium consists of quantities (x_1, x_2^h, x_2^l), asset price ψ, inflation rate μ, and multipliers $(\lambda_1, \lambda_2, \lambda_3)$ which solve equations (42)-(47),(50),(52).*

Since we have eight unknown variables with only seven equations in the Definition 2, in order to determine an equilibrium one of the two variables, inflation rate μ and the price of government bonds z, is required to be a policy variable. I assume that the central bank chooses the nominal interest rate target, $\frac{1}{z} - 1$, and implements open market operations to achieve its goal in the model. Note that the nominal interest rate of government bonds cannot be negative, i.e. $z \leq 1$, in equilibrium.

4.1 No Bank Capital Requirements

In this subsection I describe the equilibrium cases without bank capital requirements as a benchmark and explain how bank capital holdings can be changed by monetary policy. Suppose that there is no bank capital requirements, i.e. $\delta = 0$. I also assume that the supply of public assets is not restricted to support ρ portion of currency transactions,

$$V \geq \rho x^*. \tag{53}$$

We have three equilibrium cases which are similar to the regions we have studied in the previous section. If private assets are plentiful in the economy with

$$V + \beta(\psi^f + y^l) \geq x^* \tag{54}$$

then the first-best allocation, the Friedman rule equilibrium allocation, is achieved with $x_1 = x_2^l = x_2^h = x^*$, $\mu = \beta$, and $\psi = \psi_f$ which corresponds to the region 1 equilibrium in the previous section. If private assets are scarce, (54) violates and in equilibrium both the incentive constraint for currency transactions (45) and the incentive constraint for the state l (47) bind while the incentive constraint for the state h (46) may bind or not. If (46) does not bind then we have the region 2 equilibrium allocations with $x_1 < x^*$, $x_2^l < x_2^h = x^*$, $\mu > \beta$ and $\psi > \psi_f$. If (46) binds then we have the region 3 equilibrium allocation with $x_1 < x^*$, $x_2^l < x_2^h < x^*$, $\mu > \beta$ and $\psi > \psi_f$. Note that if (46) does not bind, given $\delta = 0$ bank capital is strictly positive in region 2 from the binding participation constraint (35). For the same reason bank capital is zero in region 3 when (46) binds.

In order to know how these equilibrium cases are associated with monetary policy I characterize equilibrium in region 2 and regions 3 as follows. The first-order conditions, (42) and (43), can be reduced to

$$zu'(x_1) = \pi u'(x_2^h) + (1-\pi)u'(x_2^l) \tag{55}$$

and binding constraints (45) and (47), the first-order condition for the private assets (44), and the market clearing conditions (50) and (52) can be transformed into a form of incentive constraint,

$$\begin{aligned} \rho x_1 u'(x_1) \quad + \quad (1-\rho)\pi x_2^h u'(x_2^h) \quad &+ \quad (1-\rho)(1-\pi)x_2^l u'(x_2^l) \\ = \quad \underbrace{V}_{public\,asset} \quad &+ \quad \underbrace{\frac{\beta y^h \pi u'(x_2^h) + \beta y^l (1-\pi) u'(x_2^l)}{1-\beta\{\pi u'(x_2^h) + (1-\pi)u'(x_2^l)\}}}_{private\,asset} \end{aligned} \tag{56}$$

Note that if (46) does not bind then (55) and (56) can be rewritten by plugging $x_2^h = x^*$ into the equations. If (46) binds, from (46) and (47) we have another equilibrium condition,

$$\beta(y^h - y^l) = (1-\rho)(x_2^h - x_2^l) \tag{57}$$

which can be simplified to

$$x_2^h = x_2^l + \alpha \tag{58}$$

where α is defined as $\alpha \equiv \frac{\beta(y^h - y^l)}{1-\rho}$. By plugging (58) into the first-order condition (55) and the incentive constraint (56) we can describe the equilibrium allocation (x_1, x_2^l) with two curves as shown in Figure 6. In the *FOC* curve (55) x_2^l is strictly increasing in x_1 while in the *IC* curve (56) x_2^l is strictly decreasing in x_1 because $-x\frac{u''(x)}{u'(x)} < 1$. Note that x_2^h can be also indicated in the plane since the distance between x_2^h and x_2^l is fixed as α when (46) binds. So there is a threshold point $(\tilde{x}_1, \tilde{x}_2)$ on the IC curve where the incentive constraint for the state h (46) binds at $x_1 > \tilde{x}_1$ because x_2^l is decreasing in x_1 in (56). Thus for $x_1 \leq \tilde{x}_1$ the equilibrium allocation is determined with $x_2^h = x^*$ so that this area corresponds to the region 2. For $x_1 > \tilde{x}_1$ the equilibrium allocation is determined with the binding constraint (46) with $x_2^h < x^*$ so that this area corresponds to region 3 as shown in Figure 5.

[Figure 6 here]

Given the nominal interest rate target, $\frac{1}{z} - 1$, the equilibrium allocation (x_1,x_2^l) is uniquely determined from (55)-(56) and x_2^h is passively derived by (58). As the nominal interest rate decreases, the FOC curve shifts rightward so that x_1 increases while x_2^h and x_2^l weakly decrease until it arrives at the zero-lower-bound with $z = 1$. This mechanism can be explained as follows: when the central bank injects currency and absorbs government bonds to lower the nominal interest rate, the real return on government bonds falls because the inefficiency in the credit arrangement increases by the less-supplied government bonds, x_2^h and x_2^l, respectively. Meanwhile, the real return on currency, i.e. the inverse of the inflation rate, must increase because people need to hold more real balance of currency, x_1, in equilibrium.

Given this mechanism of open market operations, when the nominal interest rate falls, bank capital in equilibrium decreases in region 2 and remains zero in region 3. Since both incentive constraints (45) and (47) bind, bank capital is just derived from the participation constraint (35) in equilibrium as $\pi\{\frac{\beta}{\mu}b + \beta(\psi + y^h) - (1-\rho)x_2^h\}$ which can be reduced to

$$\pi\{\beta(y^h - y^l) + (1-\rho)(x_2^l - x_2^h)\} \tag{59}$$

by using (47). Note that bank capital described in (59) strictly declines as x_2^l decreases while x_2^h is fixed as x^* in region 2. Then it becomes zero in region 3 because (58) holds in equilibrium. Thus as the nominal interest rate goes to the zero-lower-bound, the bank capital weakly decreases in equilibrium. It is because as the nominal interest rate falls, the assets which support credit arragements become more scarce and so the backed assets become scarce in the state h as well. Also note that if the aggregate risk becomes large, then the area of region 3 shrinks while the area of region 2 expands.

4.1.1 Asset Yields and Liquidity Premium

The real interest rate on government bonds in equilibrium can be derived from the first-order condition for government bonds (43). It is divided into the fundamental yield and the liquidity premium as

$$r^b = \frac{1-\beta\{\pi u^{'}(x_2^h)+(1-\pi)u^{'}(x_2^l)\}}{\beta\{\pi u^{'}(x_2^h)+(1-\pi)u^{'}(x_2^l)\}} = \underbrace{\{\frac{1}{\beta}-1\}}_{fundamental} - \underbrace{\frac{\{\pi u^{'}(x_2^h)+(1-\pi)u^{'}(x_2^l)\}-1}{\beta\{\pi u^{'}(x_2^h)+(1-\pi)u^{'}(x_2^l)\}}}_{liquidity\,premium} \tag{60}$$

where the liquidity premium is defined as the difference between the real interest rate on the asset and the fundamental yield from their payoffs. For strictly positive liquidity premium it is necessary to have an inefficiency of credit trade in at least one state. If the trade is efficient in the *DM*, the consumption is maximized with x^* where $u^{'}(x^*) = 1$. Thus in the term of liquidity premium in (60), $u^{'}(x_2^i) > 1$ reflects an inefficiency of credit arrangement in the state i because the incentive constraint for the state i, (46) or (47), binds. Thus as the assets are more scarce in an economy the liquidity premium on the assets rises. Note that the liquidity premium on government bonds also reflect the inefficiencies in the both states because the price of the government bonds are determined before the next period return is realized.

From the first-order condition for private assets (44), the expected real yields on private assets can be derived as

$$r^a = \frac{\bar{y}}{\psi} = \frac{1-\beta\{\pi u^{'}(x_2^h)+(1-\pi)u^{'}(x_2^l)\}}{\beta\{\pi u^{'}(x_2^h)\frac{y^h}{\bar{y}}+(1-\pi)u^{'}(x_2^l)\frac{y^l}{\bar{y}}\}}. \tag{61}$$

Note that the rate of return on government bonds can be different with the rate of return on private assets because the denominators are different in (60) and (61). Since buyers are risk-neutral with respect to the payoff in the *CM*, the fundamental yields from the payoffs of government bonds and private assets are same. This difference in the rates of return on those assets is generated by the difference in their liquidity premium. Let me define a liquidity-risk premium as a proportional difference in the rates of return on two assets which is described as

$$\frac{r^a - r^b}{r^b} = \frac{\beta\{\pi u^{'}(x_2^h)+(1-\pi)u^{'}(x_2^l)\}}{\beta\{\pi u^{'}(x_2^h)\frac{y^h}{\bar{y}}+(1-\pi)u^{'}(x_2^l)\frac{y^l}{\bar{y}}\}}. \tag{62}$$

There are two necessary conditions to have a strictly positive liquidity-risk premium. One is an inefficiency of trade in a state i with $u^{'}(x_2^i) > 1$ which generates the liquidity premium on the prices of both assets. If credit arrangements in both states are efficient then the liquidity-risk premium is zero because the fundamental yields on both assets are same. The other is aggregate risk, i.e. $y^h > y^l$, which provides more weights on the inefficiency of the state h in the denominator than in the numerator in the liquidity-risk premium in (62).[16] If there is no aggregate risk with $y^h = y^l$ then the two expected rates of return are same and the liquidity-risk premium is zero in (62). Note that we have $r^b < r^a$ in (62) as long as $x_2^l < x_2^h$ holds in equilibrium. It implies that given the

[16] The weight for the expected liquidity premium on private assets is $\{\pi\frac{y^h}{\bar{y}}, (1-\pi)\frac{y^l}{\bar{y}}\}$ while the weight for government bonds is $\{\pi, (1-\pi)\}$.

inefficiency of credit trade in the state i and the aggregate risk, the rate of return on private assets is greater than the rate of return on government bonds. Also, note that given the inefficiency in both states, if the aggregate risk increases then this liquidity-risk premium increases.

4.2 Bank Capital Requirements

In this subsection I consider how bank capital requirements can influence the real interest rates of government bonds and private assets. Moreover, I study when bank capital requirements is welfare-improving given monetary policy fixed. Suppose that a bank capital requirement, $\delta > 0$, is imposed in the state h only. The equilibrium conditions (55)-(57) can be modified into

$$zu^{'}(x_1) = \pi\{(1-\delta)u^{'}(x_2^h) + \delta\} + (1-\pi)u^{'}(x_2^l), \tag{63}$$

$$\begin{aligned} &\rho x_1 u^{'}(x_1) + (1-\rho)\pi x_2^h\{u^{'}(x_2^h) + \tfrac{\delta}{1-\delta}\} + (1-\rho)(1-\pi)x_2^l u^{'}(x_2^l) \\ &= V + \tfrac{\beta y^h \pi\{(1-\delta)u^{'}(x_2^h)+\delta\} + \beta y^l(1-\pi)u^{'}(x_2^l)}{1-\beta[\pi\{(1-\delta)u^{'}(x_2^h)+\delta\}+(1-\pi)u^{'}(x_2^l)]}, \end{aligned} \tag{64}$$

$$\beta(y^h - y^l) = (1-\rho)(\frac{x_2^h}{1-\delta} - x_2^l), \tag{65}$$

respectively. With $\delta > 0$, the threshold point between region 2 and 3, $\tilde{x}_1$, moves leftward because the gap between x_2^h and x_2^l in region 3 is reduced by $\delta > 0$ in (65). Thus the area of the region 3 expands while the area of the region 2 shrinks.

The IC curve (64) does not change in region 2 because the bank capital constraint (46) does not bind when δ increases. But in region 3 as δ increases, the IC curve (64) can move upwards as the consumption risk is shared so that x_2^l increases while x_2^h decreases as shown in the Figure 6. Notice that the equilibrium conditions (64)-(65) are similar to (22) and (26), respectively, except for the terms $V - \rho x_1 u^{'}(x_1)$ in (64) and $1-\rho$ in (65). It implies that given x_1, two curves for x_2^h and x_2^l can become closer in region 3 as δ increases while the total feasible quantities of (x_2^h, x_2^l) decrease by the $\frac{\delta}{1-\delta}$ term in (64) by Proposition 1.

The FOC curve (63) rotates as δ increases. Since the equilibrium condition (65) can be rewritten as $x_2^h = (1-\delta)(x_2^l + \alpha)$ where $\alpha = \frac{\beta(y^h - y^l)}{1-\rho}$, the right-side of the first-order condition (63) can be transformed to $(1-\delta)u^{'}((1-\delta)(x_2^l+\alpha)) + \delta$. Then given x_1 when δ increases there is a tradeoff between an intensive margin effect in which the liquidity premium rises by the inefficiency of trade by the reduced pleadgeability and an extensive margin effect in which the liquidity premium falls because the inefficiency is only applied for the pledgeable part of the assets.

***Lemma** 3. Given γ and x_1 the FOC curve rotates since if δ increases at $\delta = \tilde{\delta}$, for $x_2^l < \bar{x}_2(\tilde{\delta})$, x_2^l decreases, i.e. $\frac{\partial x_2^l}{\partial \delta} < 0$ while for $x_2^l > \bar{x}_2(\delta)$, x_2^l increases, i.e. $\frac{\partial x_2^l}{\partial \delta} > 0$.*

***Proof.** Given the left side of the first-order condition (63), $zu^{'}(x_1)$, fixed, by the implicit function theorem at $\delta = \tilde{\delta}$ we have*

$$\frac{\partial x_2^l}{\partial \delta} = -\frac{\pi\{1-(1-\gamma)u'((1-\tilde{\delta})(x_2^l+\alpha))\}}{\pi(1-\tilde{\delta})^2(u''((1-\tilde{\delta})(x_2^l+\alpha))+(1-\pi)u''(x_2^l)} \gtreqless 0. \tag{66}$$

Since the denominator of (66) is strictly negative, given γ if $x_2^l < \bar{x}_2(\tilde{\delta})$ where $\bar{x}_2(\tilde{\delta})$ satisfies with $1 = (1-\gamma)u'((1-\tilde{\delta})(\bar{x}_2(\tilde{\delta})+\alpha)$, then we have $\frac{\partial x_2^l}{\partial \delta} < 0$ and otherwise $\frac{\partial x_2^l}{\partial \delta} \geq 0$. Thus the FOC curve rotates counter-clockwise with the center of $\bar{x}_2(\tilde{\delta})$■

Since currency is available to use in both currency trade and credit arrangements, given the nominal interest rate, rates of return on currency and government bonds must be equal in the first-order condition. Lemma 3 implies that when δ increases, the real interest rate on government bonds is adjusted so that the real interest rate on currency, i.e. the inverse of the inflation rate, must be also changed. Note that from (66) $\bar{x}_2(\delta)$ is strictly decreasing in γ. Thus when γ is sufficiently high, the *FOC* curve tends to shift leftward further. Also note that $\bar{x}_2(\delta)$ is strictly increasing in δ from (66).

This result has an implication on the monetary policy. Since the *FOC* curve can shift leftward by imposing bank capital requirements, the feasible set of equilibrium allocation by choosing monetary policy can be shrinked. For example, the initial allocation x_1 and the inflation rate μ, which is feasible with $z = 1$ and $\delta = 0$, can be no longer feasible with $z = 1$ and $\delta > 0$ if the *FOC* curve moves leftward.

[Figure 7 here]

4.2.1 Asset Yields and Liquidity Premium

In this subsection I consider how bank capital requirements can influence the inflation rate and real interest rates on assets in equilibrium. Let me divide the cases by the direction of x_1 in equilibrium when δ increases. Remember in region 2 the equilibrium allocation does not change until the incentive constraint for the state h binds. Thus suppose that given monetary policy fixed, the equilibrium allocation exists in region 3. When δ increases the *IC* curve (64) shifts upwards in region 3 and the *FOC* curve (63) rotates. If x_1 is maintained as before then by the first-order condition for currency trade (42) the inflation rate does not change. Since the nominal interest rate is fixed, the real interst rate on government bonds does not change either from (43) and (60). However, the real interest rate on private assets decreases because the liquidity-risk premium with $\delta > 0$ in (67) decreases. In (67) the numerator does not change since z and x_1 are maintained in the first-order condition (63). But the denominater increases because when δ is raised, x_2^l increases in equilibrium so that $u'(x_2^l)$ decreases whereas $(1-\delta)u'(x_2^h)+\delta$ increases. As a result r^b remains at its original level while r^a adjusts downward.

$$\frac{r^a - r^b}{r^b} = \frac{\beta[\pi\{(1-\delta)u'(x_2^h)+\delta\}+(1-\pi)u'(x_2^l)]}{\beta[\pi\{(1-\delta)u'(x_2^h)+\delta\}\frac{y^h}{\bar{y}}+(1-\pi)u'(x_2^l)\frac{y^l}{\bar{y}}]} \tag{67}$$

Similarly, when δ increases if x_1 is determined at the lower level of the original x_1, the inflation rate goes up so that the real return of government bonds decreases. Then the real return on private assets also decreases because both the liquidity-risk premium and the real interest rate on government bonds decreases. Finally, if x_1 is determined at the higher level of the original x_1, the inflation rate falls and the real interest rate on government bonds increases. Then the direction of the real interest rate on private assets is ambiguous because the liquidty-risk premium decreases while the real interest rate on government bonds increases.

This result implies that given monetary policy fixed, bank capital requirements can adjust real interest rates on government bonds and private asset in equilibrium. That means, bank capital requirements can also be effective at the zero-lower-bound where monetary policy is limited to lower the real interest rates further. In this respect bank capital requirements can be also considered as an unconventional policy option at the zero-lower-bound.

4.2.2 Welfare-improving Bank Capital Requirements

In this subsection I analyze when bank capital requirements will be beneficial given monetary policy fixed. As is shown in the previous section bank capital requirements are beneficial for sharing consumption risk in credit arrangements. However, in this extended model there are currency transactions as well. Thus the welfare improvement of bank capital requirements also depends on how capital requirements influence currency exchanges in the *DM*. Suppose that when δ increases, the *FOC* curve shifts leftward more than the *IC* curve shifts rightward. Then x_1 decreases and the inflation rate goes up in equilibrium. This implies that the inefficiency in the currency trade increases by imposing bank capital requirements. Thus although the credit arrangements can improve as the consumption risk is shared, the currency trade can be worse off.

***Lemma** 4. Given $\delta \geq 0$, when the allcation x_1 increases and x_2^l decreases by moving along the IC curve, the welfare improves.*

***Proof.** Given δ if the nominal interest rate decreases then the equilibrium allocation moves along the IC curve. Thus I consider that given δ and the nominal interest rate, $\frac{1}{\hat{z}}-1$, the allocation is welfare-improving as $\hat{z}$ increases. If we add up the expected utilities across agents in a stationary equilibrium, the welfare measure in the extended model is described as*

$$W = \rho\{u(x_1) - x_1\} + (1-\rho)\pi\{u(x_2^h) - x_2^h\} + (1-\rho)(1-\pi)\{u(x_2^l) - x_2^l\} + \bar{y} \tag{68}$$

that represents the sum of surpluses from trade in the DM. Suppose that there exists a unique equilibrium in region 3 given the nominal interest rate, $\frac{1}{\hat{z}} - 1$. In region 3 since the incentive constraint for state h also binds, from (63) and (65) we have the modified first-order condition,

$$\hat{z}u^{'}(x_1) = \pi\{(1-\delta)u^{'}((1-\delta)(x_2^l+\alpha)) + \delta\} + (1-\pi)u^{'}(x_2^l), \tag{69}$$

and from (64) and (65) the modified equilibrium condition,

$$V + K(x_2^l) = \rho x_1 u^{'}(x_1) + (1-\rho)\pi x_2^h\{u^{'}(x_2^h) + \tfrac{\delta}{1-\delta}\} + (1-\rho)(1-\pi)x_2^l u^{'}(x_2^l) \tag{70}$$

where $K(x_2^l) = \frac{\beta y^h \pi\{(1-\delta)u^{'}((1-\delta)(x_2^l+\alpha))+\delta\}+\beta y^l(1-\pi)u^{'}(x_2^l)}{1-\beta\{\pi\{(1-\delta)u^{'}((1-\delta)(x_2^l+\alpha))+\delta\}+(1-\pi)u^{'}(x_2^l)\}}$. *In the* (x_1, x_2^l) *plane the slope of welfare function (68) with* $x_2^h = (1-\delta)(x_2^l+\alpha)$ *at* $z = \hat{z}$ *is*

$$\frac{\partial x_2^l}{\partial x_1} = -\frac{\rho\{u^{'}(x_1)-1\}}{(1-\rho)[\pi\{(1-\delta)u'((1-\delta)(x_2^l+\alpha)) - (1-\delta)\} + (1-\pi)(u'(x_2^l)-1)]} = -\frac{\rho}{(1-\rho)\hat{z} - \frac{(1-\hat{z})(1-\rho)}{u^{'}(x_1)-1}} \tag{71}$$

while the slope of the equilibrium condition (70) at $z = \hat{z}$ *is*

$$\frac{\partial x_2^l}{\partial x_1} = -\frac{\rho(1-\gamma)u^{'}(x_1)}{(1-\rho)(1-\gamma)[\pi\{(1-\delta)u^{'}((1-\delta)(x_2^l+\alpha))+\delta\}+(1-\pi)u^{'}(x_2^l)]+(1-\rho)\pi\gamma\delta-K^{'}(x_2^l)} = -\frac{\rho}{(1-\rho)\hat{z}+\frac{(1-\rho)\pi\gamma\delta-K^{'}(x_2^l)}{u^{'}(x_1)-1}}. \tag{72}$$

Then the slope of welfare function is steeper than the slope of the equilibrium condition (70) since $K^{'}(x_2^l) < 0$. *Thus given bank capital requirements, the welfare improves when the allocation* x_1 *increases and* x_2^l *decreases along the IC curve*■

Lemma 4 shows that the welfare improves as the allocation x_1 increases and x_2^l decreases along the IC curve. Thus we can divide the effect of bank capital requirements on the equilibrium allocation into two different factors. One is risk-sharing effect by which the allocation in the point A moves to the allocation in the point B in Figure 7. The other is illiquidity effect by which the allocation in the point B moves to the allocation in the point C in Figure 7. Thus by the illiquidity effect, a higher quantity of goods is traded in credit arrangements while a lower quantity of goods is traded in currency transactions. Note that the risk-sharing effect improves welfare, but the illiquidity effect on the welfare is ambiguous because the direction of the new equilibrium allocation depends on the degree of the shifts in both FOC and IC curves.

Proposition *2. If agents are sufficiently risk-averse with* $\gamma > \gamma^*$ *and the equilibrium allocation* x_1 *increases by* δ *in a neighborhood of* $\delta = 0$, *then the welfare improves by imposing* $\delta > 0$.

Proof *Given the risk-aversion of agents* $\gamma > \gamma^*$ *and the nominal interest rate* $\hat{z}$, *suppose that equilibrium allocation* $(\hat{x}_1, \hat{x}_2^l)$ *exists in region 3. Define an equilibrium allocation as* $(\tilde{x}_1, \tilde{x}_2^l)$ *which is determined with* $\delta = \tilde{\delta} > 0$. *Then the movement from* $(\hat{x}_1, \hat{x}_2^l)$ *to* $(\tilde{x}_1, \tilde{x}_2^l)$ *is divided into two parts. One is the movement of* x_2^l *along the vertical line at* $x_1 = \hat{x}_1$. *The other is the movement that* x_1 *increases and* x_2^l *decreases along the changed IC curve. The welfare improves from the first movement by Propostion 1 and also improves from the second movement by Lemma 4* ■

The Proposition 2 implies that if the sufficient condition for Proposition 1, $\gamma > \gamma^*$, is satisfied and currency transactions increase by the shift of the FOC curve then bank capital requirements can improve welfare as a sufficient condition. Note that as long as the benefit of sharing consumption risk is greater than the cost of holding additional capital and the addtional inefficiency in currency exchange, bank capital requirements are beneficial for society. Thus even though x_1 decreases by imposing bank capital requirements, the welfare can improve if the risk-sharing effect dominates the illiquidity effect.

Notice that this illiquidity effect in which the allocation moves along the IC curve is similar to the effect of open market operations because the quantities of currency trade and credit arrangements are adjusted. However, it is different because this illiquidity effect is generated by affecting prices through capital requirements instead of changing the supply of liquid and illiquid assets through open market operations. This result implies that bank capital requirements can also function as a monetary policy tool without exchanging liquid and illiquid assets.

These results are also shown in Figure 8 with numerical examples. In the right-side panel graphs given zero nominal interest rate where the conventional monetary policy is restricted, imposing capital requirements can reduce real interest rates on assets further as we discussed. Note that the difference between real returns on government bonds and on private assets is decreasing in δ since consumption risk is shared in credit arrangements. In this case the welfare improves as δ increases although x_1 decreases. It is because the benefit of sharing risk in credit arrangements is greater than the cost of holding bank capital and the inefficiency in currency exchange. With the middle and the left panel graphs, it is found additionally that bank capital requirements will be beneficial as the nominal interest rate approaches to zero. As shown in the upper panel graphs currency exchange, x_1 , does not change by imposing capital requirements in each case. However, the cost of holding additional bank capital can be different by cases. Since total assets become less scarce when the nominal interest rate approaches to zero by Lemma 4, the cost of holding bank capital is decreasing in z.[17] Thus in this numerical example bank capital requirements will be beneficial as the nominal interest rate approaches to zero because the cost of holding bank capital is decreasing in z.

[Figure 8 here]

5 Conclusion

I construct a banking model in which a contingent deposit contract is chosen to provide liquidity efficiently given aggregate risk. With limited commitment, deposit claims are backed by bank

[17] In monetary equilibrium the cost of holding assets does not depend only on the level of the real interest rate on private assets because bankers also hold currency for their asset portfolio. In this case the cost of holding currency is lower as the nominal interest rate goes to zero since the level of currency exchange, x_1, is increasing in z.

assets, so that a liquidity premium on assets can arise when the supply of assets is insufficient for efficient exchange in at least one state. In this case it is costly to hold bank capital, but by holding bank capital the banker can manage liquidity for the depositors in an efficient way. A pro-cyclical bank capital requirement, which forces bankers to hold additional bank capital in the high-return state, can improve welfare by smoothing consumption. Although it is costly to hold additional bank capital, reducing the pledgeability of the assets in the high-return state can relax the incentive constraint in the low-return state by affecting the asset price. In the extended model with money and government bonds, the relationship between bank capital requirements and monetary policy is studied. Since bank capital requirements adjust the pledgeability of the assets, bank capital requirements can influence macroeconomic variables such as real interest rates on the assets and the inflation rate. If agents are risk-averse enough, bank capital requirements will be beneficial as the nominal interest rate approaches to zero.

This paper takes steps to understand the role of bank capital for efficient liquidity provision. It also sheds light on the rationale for bank capital requirements as a macro-prudential policy that accommodates risk-sharing by affecting the pledgeability of assets. This implication is consistent with recent empirical studies in which macro-prudential policy tools are shown as effective in stabilizing credit-cycles. Lim et al. (2011) find that several macro-prudential tools, such as the Loan-to-Value ratio cap, dynamic provisioning, and the counter-cyclical buffer, can reduce the pro-cyclicality of credit growth by using the 2011 IMF survey data. Akinci and Olmstead-Rumsey (2015) develop a new index of macro-prudential policies in 57 countries and show that macro-prudential policy variables exert a negative effect on bank credit growth with a dynamic panel data model. However, this result cannot address an answer for welfare issues because the cost of externality is given exogenously in the previous theoretical models. This paper can contribute to this growing literature by providing a relevant justification for welfare improvement with a theoretical model in which the cost of holding capital is endogenously chosen.

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Figure 1. Time line

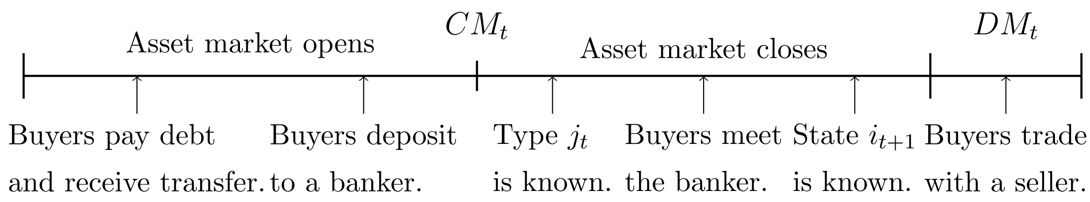

Figure 2. Regions with No Bank Capital Requirements($\delta^h = \delta^l = 0$)

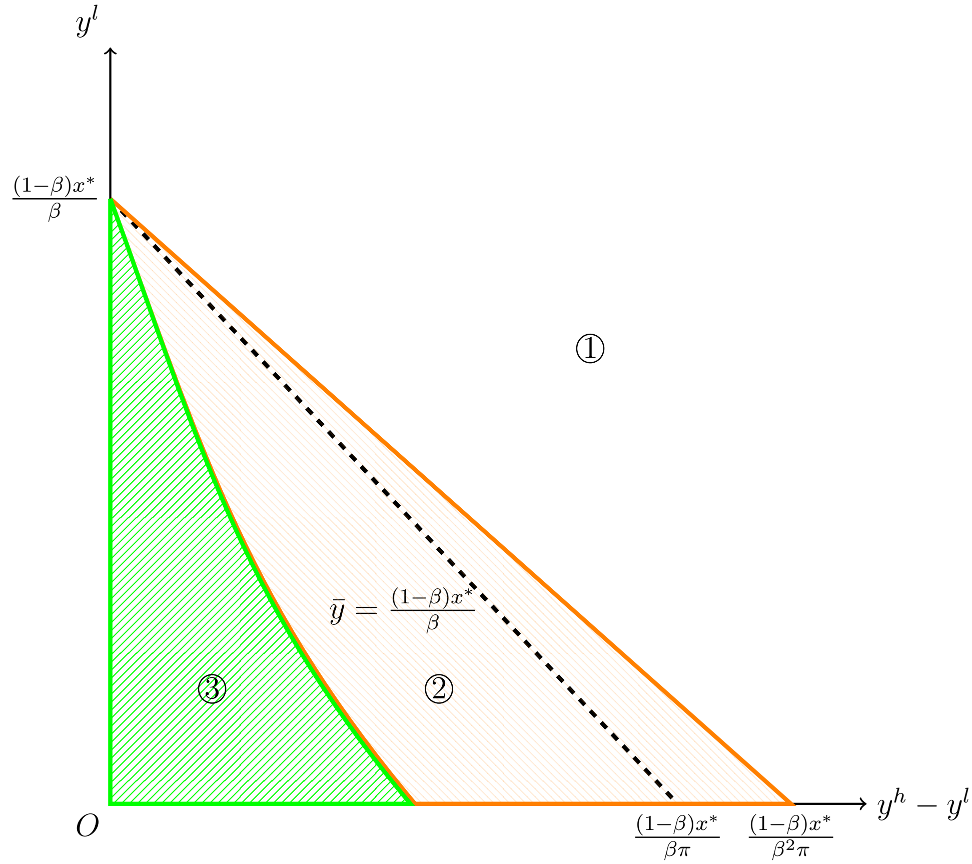

Figure 3. Risk-sharing with Bank Capital Requirements

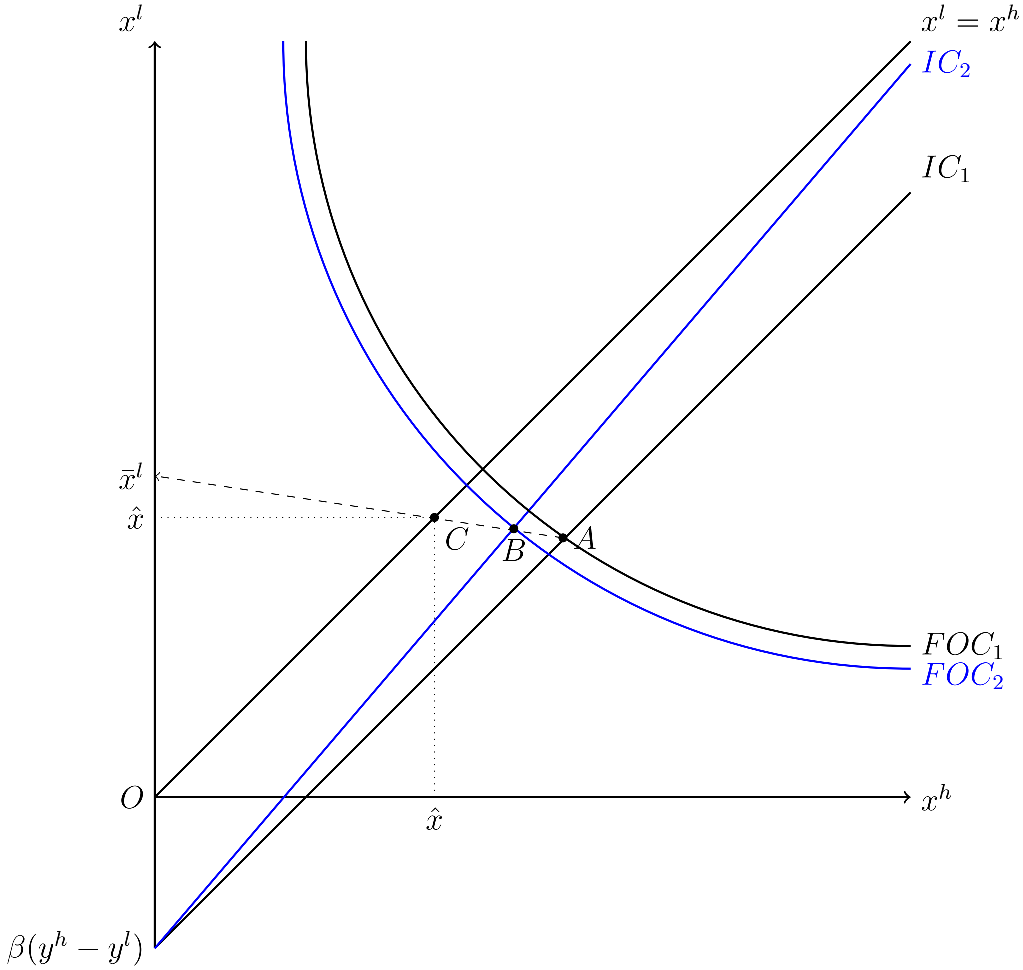

Figure 4. Regions of Welfare Improvement by Risk-Aversion($\pi = 0.5, \beta = 0.8, x^* = 1$)

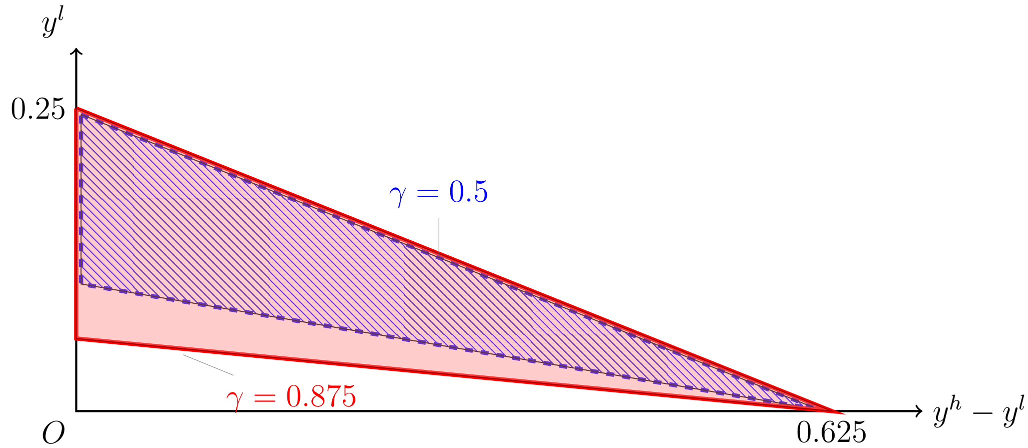

Figure 5. Welfare Improvement by Bank Capital Requirements

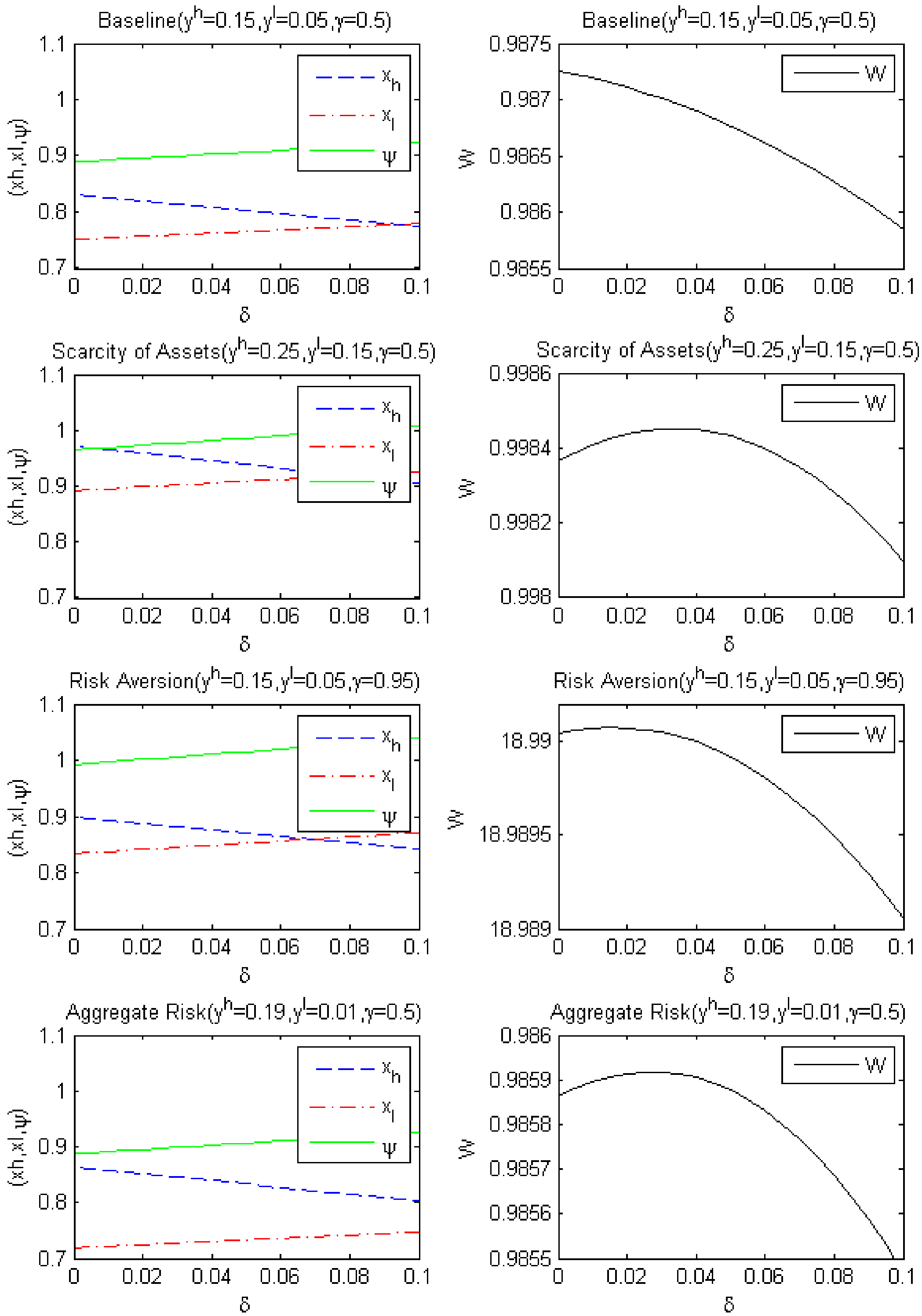

Figure 6. Monetary Equilibrium

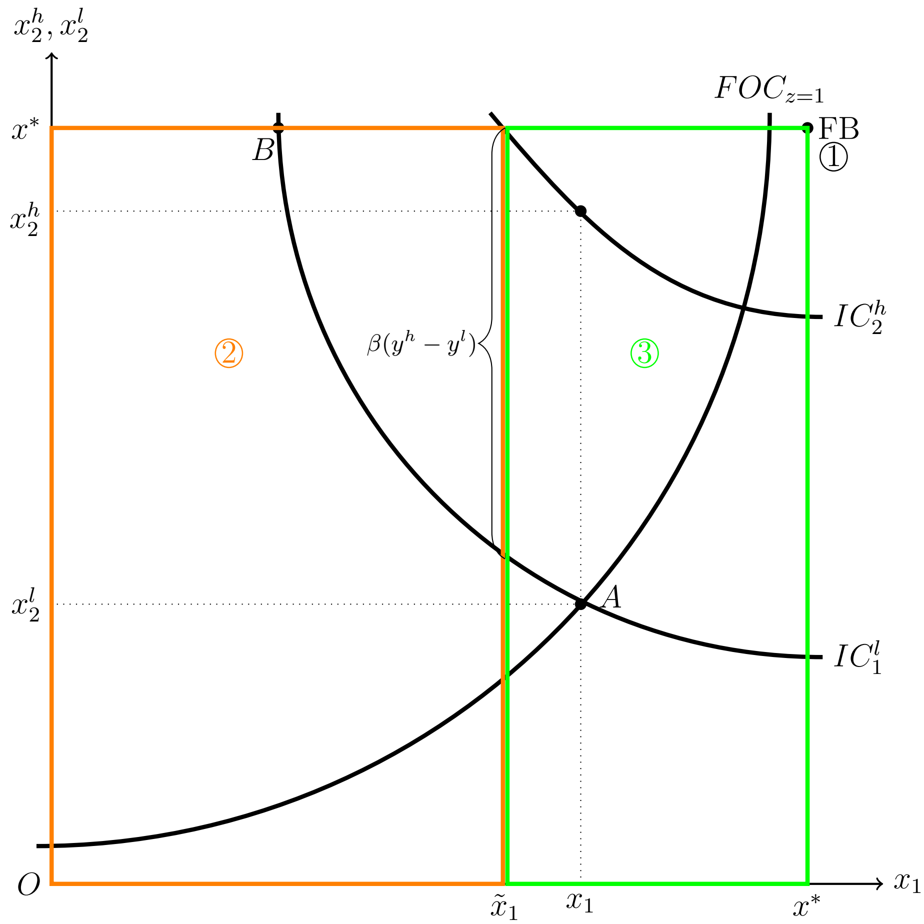

Figure 7. Monetary Equilibrium with Bank Capital Requirements($\delta > 0$)

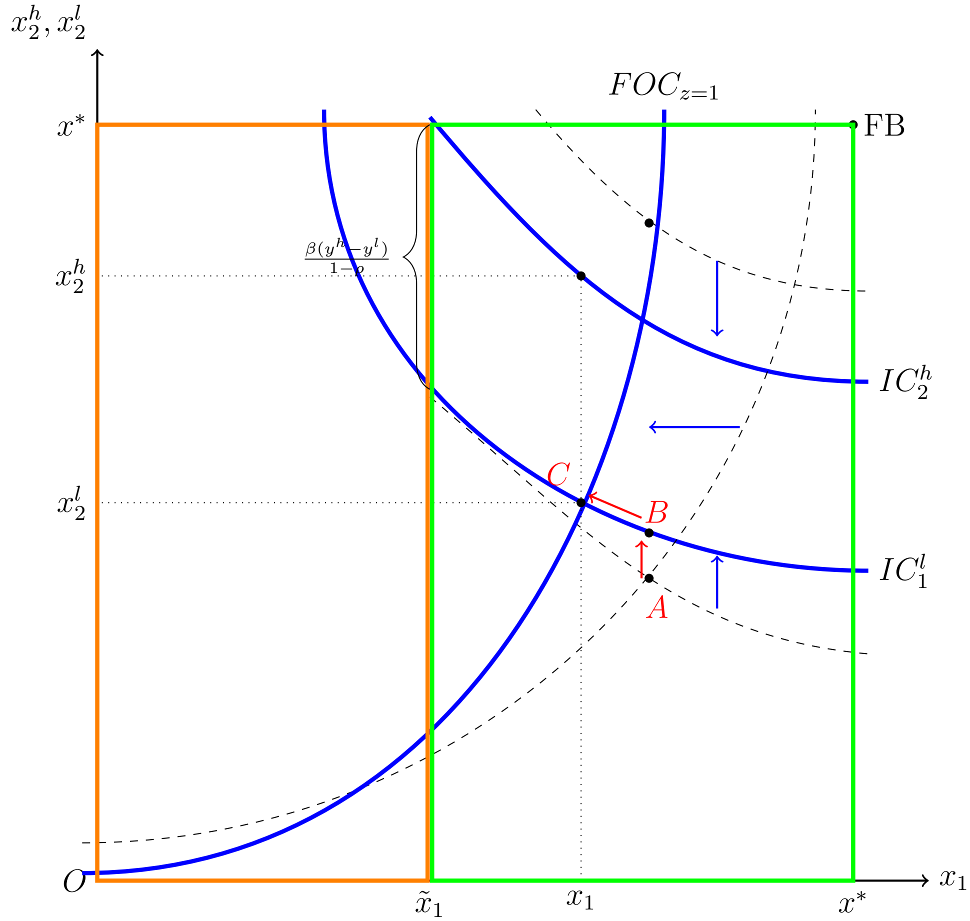

Figure 8. Welfare Improvement of Bank Capital Requirements in Monetary Equilibrium